$$\tau = I\frac{d^2\theta}{dt^2}$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0 \ \ldots\ldots\ldots (i)$$

This is the equation of simple angular harmonic motion.

Hence, the motion of torsional pendulum is angular harmonic.

Angular Frequency $\omega = \sqrt{\frac{C}{I}}$

and $\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$

This is the required expression for time period of torsional pendulum.

OR,

Derive the differential equation of the forced oscillation of LCR circuit with an AC source and find the expression for the current amplitude. Hence explain the condition of current resonance in such circuit.

⇒ *Refer to Q.N. 2 of 070 Bhadra*

2. **A 750g block oscillates on the end of a spring whose force constant, k = 56N/m. The mass moves in a fluid which offers a resistive force F = -bv, where b = 0.162Ns/m. What is the period of the oscillation?**

⇒ The period of oscillation is given as

$$T = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

Here, k = 56 N/m, m = 0.75 kg, b = 0.162 Ns/m

$\therefore$ T = 0.7268 sec

3. **A room has dimensions 6m×4m×5m. Find:**
   i. **Mean free path of sound wave in the room**
   ii. **The number of reflections made per second by the sound wave with the walls of the room. (Take velocity of sound in air = $350ms^{-1}$).**

⇒ Volume of room (V) = 6m × 4m × 5m = $120m^3$

i. Mean free path is the average distance covered by a sound wave through air between two consecutive reflections with the walls of the room.

$$\text{i.e., } \sigma = \frac{4 \times \text{volume}}{\text{Area}}$$

$$\begin{aligned}\text{Total surface area} &= 2(lb + bh + lh)\\ &= 2(6 \times 4 + 4 \times 5 + 6 \times 5)\\ &= 148m^2\end{aligned}$$

$$\therefore \sigma = \frac{4 \times 120}{148} = 3.242 \text{ m}$$

ii. Number of reflections made per second by sound wave is

$$\begin{aligned} n &= \frac{\text{velocity}}{\text{mean free path}}\\ &= \frac{350}{3.243}\\ &= 108\end{aligned}$$

4. **Define interference. Show that interference in thin film due to reflected and transmitted lights are complementary.**

⇒ **Interference**

The modification of distribution of energy due to superposition of two light waves is called interference of light.

Interference in thin film due to reflected light

Consider a thin film of thickness t and refractive index μ. A ray of light XA is partially reflected along AT and partially refracted along AB. At B, light is reflected along BC and emerges out along CY.

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1. Define free oscillation and write two differences between compound and torsion pendulum. Show that time period of compound pendulum is minimum when $k = l_1$.

⇒ **Free oscillation:**

The motion in which particle moves without any restriction of the force i.e., motion of the particle is not retarded by external force such as friction etc. is called free oscillation. For example: motion of electrons around nucleus, motion of simple pendulum in vacuum, etc.

Compound pendulum	Torsion pendulum
1. In compound pendulum, amplitude of oscillation is assumed to be small.	1. In torsion pendulum, amplitude of oscillation is not assumed to be small.
2. This pendulum can be used to measure moment of inertia, acceleration due to gravity, etc.	2. This pendulum can be used to measure the rigidity of wire.

Time period of compound pendulum is given by

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l_1} + l_1}{g}}$$

where k is radius of gyration, and l_1 is the distance of point of suspension from centre of gravity.

The time period will be minimum if $\frac{k^2}{l_1} + l_1$ is minimum.

So, differentiating $\frac{k^2}{l_1} + l_1$ with respect to l_1

$$\frac{d}{dl_1}\left[\frac{k^2}{l_1} + l_1\right] = 0 \quad \Rightarrow \frac{-k^2}{l_1^2} + 1 = 0$$

or, $\frac{k^2}{l_1^2} = 1$

$k^2 = l_1^2$

$\therefore k = l_1$ proved.

OR,

Why are the electromagnetic oscillations always damping? Explain how can we keep the oscillations continue? Derive the differential equation of the damped electromagnetic oscillation and write its solution.

⇒ Practically, all oscillations are damping. In case of electromagnetic oscillation, the motion is affected by field produced by oscillator such as electric field and magnetic field, heat produced in resistance, etc. So, these motions are usually damping.

To keep the oscillation continue, we can apply the periodic external force with definite period and frequency. Such an oscillation is called forced vibration.

For next part, refer to Q.N. 1 of 069 Ashad

2. **A wave of frequency 500Hz has a phase velocity of 200m/s. (i) How far apart are two points 30° out of phase? (ii) What is the phase difference between two displacements at a point at times 10^{-3}s apart?**

⇒ Here, frequency of wave (f) = 500 Hz

Phase velocity (v) = 200 m/s

$$v = \lambda . f \Rightarrow \lambda = \frac{v}{f} = \frac{200}{500} = 0.4 \text{ m}$$

$$\text{Time period (T)} = \frac{1}{f} = \frac{1}{500} = 2\times10^{-3} \text{ sec.}$$

i. A cycle is equivalent to 360° so that 30° corresponds to $\left(\frac{1}{12}\right)^{th}$ of a cycle.

$\therefore$ Corresponding length is $\frac{\lambda}{12} = \frac{0.4}{12} = \frac{1}{30} \text{ m} = 0.033 \text{ m}$

ii. Here the time 10^{-3} sec is half of time period $T = 2\times10^{-3}$ sec and corresponds to half of one cycle or half of 360°. Thus, the phase difference is 180°.

3. **Sound waves are emitted uniformly in all direction from the speaker in a large hall. Prove that the amplitude of the sound waves change with the distance r at any point from the speaker is, $a = \frac{1}{r}\sqrt{\frac{P}{2\pi\rho v\omega^2}}$. Where P is power of the sound wave moving with velocity v in the medium of density ρ and ω is the angular frequency.**

⇒ Consider a sound wave travelling along +ve x-direction is

$y = A \sin(\omega t - kx)$ .....(i)

$$\text{Velocity, } v = \frac{dy}{dt} = a\cos(\omega t - kx) . \omega = \omega a \cos(\omega t - kx)$$

Total energy of particle is

$$E = K.E + P.E.$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}ky^2 \; [\because k = m\omega^2]$$

$$= \frac{1}{2}m\{\omega a\cos(\omega t - kx)\}^2 + \frac{1}{2}m\omega^2\{a\sin(\omega t - kx)\}^2$$

$$= \frac{1}{2}m\omega^2 a^2\{\cos^2(\omega t - kx) + \sin^2(\omega t - kx)\}$$

$$\therefore E = \frac{1}{2}m\omega^2 a^2 \text{........(ii)}$$

If there are 'n' identical particles per unit volume, then energy density is given by

$$U = nE = \frac{1}{2}(nm)\omega^2 a^2 = \frac{1}{2}\rho\omega^2 a^2 \; [\rho = nm \text{ is density}]$$

If we consider a tube of cross section S and length L, then total energy within the tube is

$$W = UV = \frac{1}{2}\rho\omega^2 a^2 SL \text{ .... (iii)}$$

[V = SL is volume of the tube]

Intensity due to wave is defined as the flow of energy per unit area per second.

$$I = \frac{W}{St} = \frac{\frac{1}{2}\rho\omega^2 a^2 SL}{St}$$

$$= \frac{1}{2}\rho\omega^2 a^2 v \text{ ....(iv)} \qquad [\because v = \frac{L}{t}]$$

Since the sound waves are emitted uniformly in all direction from the speaker, the expression for spherical wave is

$$I = \frac{P}{4\pi r^2} \text{ ....(v)}$$ where P is power of sound.

$$\text{or, } \frac{1}{2}\rho\omega^2 a^2 v = \frac{P}{4\pi r^2}$$

$$\text{or, } a^2 = \frac{P}{2\pi r^2 v\rho\omega^2}$$

$$\boxed{\therefore a = \frac{1}{r}\sqrt{\frac{P}{2\pi\rho v\omega^2}}}$$ Proved.

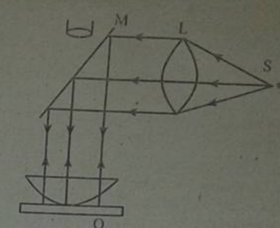
4. **Define the diffraction of light. Show that the intensity of second primary maxima is 1.62% of central maxima in Fraunhoffer's single slit diffraction.**

⇒ *Refer to Q.N. 4 of 070 Ashad*

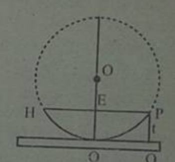
5. **What are Newton's rings? How are they formed? Show that the diameter of dark Newton's rings by reflected system of light are proportional to the square root of natural number.**

⇒ Circular interference fringes are produced by enclosing thin air film of varying thickness between plane glass plate and plano-convex lens of large radius of curvature; such rings are called Newton's ring. For monochromatic light, dark and bright fringes are produced whereas for white light, colored fringes are observed.

Consider a monochromatic beam of light produced by source S is incident on mirror M which reflects it towards plano-convex lens and forms a interference pattern which can be observed by microscope.



Let R be the radius of curvature of lens and t is the thickness of air film.



For dark fringe due to reflected light system,

Path difference = $n\lambda$

or, $2\mu t \cos r = n\lambda$

For small angle, $\cos r \approx 1$ and for air film, $\mu = 1$.

$\therefore 2t = n\lambda$ ....(1)

From geometry, HE. EP = OE (2R − OE)

$r.r = t.2R$ $\qquad [\because 2R - OE \approx 2R$ approximately]

$r^2 = 2tR$ $\qquad$ where r is the radius of n^{th} dark ring

$t = \frac{r^2}{2R}$ ......(2)

From (1) & (2),

$2.\frac{r^2}{2R} = n\lambda$

or, $r^2 = n\lambda R$

or, $D^2 = 4n\lambda R$; D is diameter of n^{th} dark ring

or, $D_n = \sqrt{4n\lambda R}$

Hence, $D_n \propto \sqrt{n}$ proved.

OR,

Differentiate between quarter wave plate and half wave plate. Use the reference of double refraction to describe how you distinguish positive and negative crystal. [Describe with a neat diagram].

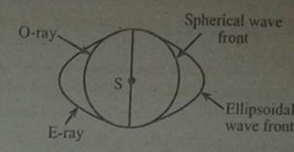
⇒

Quarter wave plate	Half wave plate
1. It produces phase difference of $\frac{\pi}{2}$ and path difference of $\frac{\lambda}{4}$ in between O–ray and E–ray.	1. It produce phase difference of π and path difference of $\frac{\lambda}{2}$ in between E–ray and O–ray.
2. Thickness of quarter wave plate is given by $t = \frac{\lambda}{4(\mu_0 - \mu_E)}$	2. Thickness of half wave plate is given by $t = \frac{\lambda}{2(\mu_0 - \mu_E)}$

A point source of light in double refracting crystal is the origin of two wavefronts. For ordinary ray, wave front is spherical because light travels with equal velocity in all direction. For extra-ordinary ray, the wave front is ellipsoidal because velocity of light is different in different direction. Along optic axis, velocity of O–ray and E–ray are equal.

Negative crystal

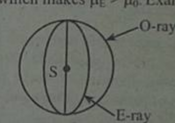
Ordinary wavefront lies inside the extra-ordinary wave front. Here, velocity of E–ray is greater than O–ray which makes $\mu_0 > \mu_E$ .Example: calcite crystal.



Negative Crystal

Positive crystal

For positive crystal, ordinary wavefront lies outside the extra- ordinary wave front. Here ,velocity of O–ray is greater than E– ray which makes $\mu_E > \mu_0$. Example: quartz crystal.

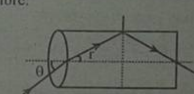


Positive crystal

6. **Define acceptance angle in optical fiber. Show that, numerical aperture (NA) = $\mu_1\sqrt{2\Delta}$; where μ_1 is the refractive index of core of optical fiber, Δ is fractional refractive index change.**

⇒ **Acceptance Angle**

It is defined as the maximum angle that a light ray can have relative to the axis of the fibre which will propagate light ray down the fibre.



The numerical aperture is defined as the sine of the acceptance angle. So, we may write

$$NA = \sin\theta_0$$

$$NA = \sqrt{\mu_1^2 - \mu_2^2}$$

where μ_1 & μ_2 are refractive index of core and cladding respectively.

$$NA = \sqrt{\left(\frac{\mu_1 + \mu_2}{2}\right)\left(\frac{\mu_1 - \mu_2}{\mu_1}\right) \times 2\mu_1}$$

Approximating $\frac{\mu_1 + \mu_2}{2} \approx \mu_1$

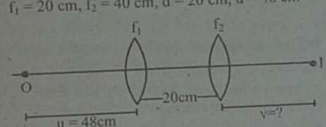
$NA = \sqrt{2\mu_1^2 \Delta}$, where $\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$ is fractional change

In refractive index.

$\boxed{\therefore NA = \mu_1\sqrt{2\Delta}}$ proved.

7. **Two thin converging lenses of focal lengths 20cm and 40cm are placed co-axially 20cm apart. An object is located at a distance of 48cm from first lens. Find the positions of principal points and image.**

⇒ $f_1 = 20$ cm, $f_2 = 40$ cm, $d = 20$ cm, $u = 48$ cm



Focal length of equivalent lens is

$$f = \frac{f_1 . f_2}{f_1 + f_2 - d} = \frac{20 \times 40}{20 + 40 - 20} = 20 \text{ cm}$$

Position of principal points are

$$\alpha = \frac{f.d}{f_2} = \frac{20 \times 20}{40} = 10 \text{ cm}$$

$$\beta = -\frac{f.d}{f_1} = \frac{20 \times 20}{20} = -20 \text{ cm}$$

Object distance of equivalent lens $U = -(u + \alpha)$
$= -58$ cm

Now,

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{V} + \frac{1}{58} = \frac{1}{20} \Rightarrow V = \frac{58 \times 20}{58 - 20} = 30.52 \text{ cm}$$

Position of image from second lens is

$$v = V + \beta = 30.52 - 20 = 10.52 \text{ cm}$$

8. **A 200mm long tube containing 48cm³ of sugar solution produces an optical rotation of 11° when placed on a polarimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of solution.**

⇒ Length of tube (L) = 200 mm = 20 cm

Volume of sugar (V) = 48 cm^3

Optical rotation (θ) = 11°

Specific rotation (S) = 66°

We have,

$$S = \frac{10\theta}{LC}$$

$$\text{or, } 66 = \frac{10 \times 11}{20 \times C}$$

$$\therefore C = \frac{1}{12} \text{ gm/cc}$$

Amount of sugar in solution (m) = CV = $48 \times \frac{1}{12} = 4$ gm

9. **Derive an expression for the electric field at point P at a distance 'z' from a circular plastic disc of radius R along its central axis. Explain what will happen to the electric field if (i) R→∞ while keeping z finite, and (ii) z→0 while keeping R finite.**

⇒ *Refer Q.N. 9 of 070 Bhadra to get expression*

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

(i) For $R \to \infty$

$$E = \frac{\sigma}{2\varepsilon_0}$$

This is the electric field produced by an infinite sheet of uniform charge located on one side of non conductor.

(ii) For $z \to 0$,

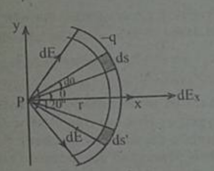
$$E = \lim_{z \to 0} \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}$$

OR,

A plastic rod contains uniformly distributed charge −q. The rod has been bent in 120° circular arc of radius r as shown in figure below. Prove that the electric intensity at the centre of the bent rod is $E = \frac{0.83q}{4\pi\epsilon_0 r^2}$.

⇒



Consider a differential element of arc length ds located at angle θ above the axis.

Electric field due to this element is

$$dE = \frac{\lambda ds}{4\pi\varepsilon_0 r^2} \quad .....(1)$$ where λ is linear charge density.

The element ds has symmetrically located element ds' in the bottom half of the rod. Electric field dE' set up at P by ds'

also has magnitude equal to dE but field vector points towards ds'.

If we resolve electric field vectors of ds & ds' into x and y components, it is seen that their y–components cancel.

Component dE_x set up by ds is

$$dE_x = dE\cos\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r^2}\cos\theta\, ds \qquad [\because ds = r d\theta]$$

$$= \frac{\lambda r}{4\pi\varepsilon_0 r^2}\cos\theta\, d\theta$$

Total field is

$$E = \int dE_x = \frac{\lambda}{4\pi\varepsilon_0 r}\int_{\theta=-60^\circ}^{\theta=60^\circ}\cos\theta\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta]_{-60^\circ}^{60^\circ}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\{\sin 60^\circ - (-\sin 60^\circ)\}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r} \times \sqrt{3}$$

$$\lambda = \frac{\text{charge}}{\text{length}}$$

$$= \frac{q}{2\pi\frac{r}{3}} = \frac{3q}{2\pi r}$$

$$\therefore E = \frac{3q}{4\pi\varepsilon_0 r}\cdot\frac{\sqrt{3}}{2\pi r} = \frac{0.83q}{4\pi\varepsilon_0 r^2}$$ proved.

10. **If a parallel plate capacitor is to be designed to operate in an environment of fluctuating temperature, prove that the rate of change of capacitance with temperature T is given by: $\frac{dC}{dT} = C\left[\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right]$ where symbols carry their usual meanings.**

⇒ Capacitance of a parallel pate capacitor is

$$C = \varepsilon_0 \frac{A}{x} \quad ....(i)$$

where A is area of plate and x is separation of plate.

Differentiating w.r.t. temperature, we get

$$\frac{dC}{dT} = \varepsilon_0 \frac{d}{dT}\left(\frac{A}{x}\right)$$

$$= \varepsilon_0 \left[\frac{\frac{dA}{dT}x - A.\frac{dx}{dT}}{x^2}\right]$$

$$= \varepsilon_0 \left[\frac{1}{x}\frac{dA}{dT} - \frac{A}{x^2}\frac{dx}{dT}\right]$$

$$= \varepsilon_0 \frac{A}{x}\left[\frac{1}{x}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right]$$

$$\boxed{\therefore \frac{dC}{dT} = C\left[\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right]}$$ proved.

11. **Calculate the relaxation time for the electrons of sodium atom. The number of atoms per cubic cm in sodium is 2.5 × 10^{22}, and the electrical conductivity is 1.9 × 10^7 S/m.**

⇒ Number density (η) = 2.5 × 10^{22}/m^3

Electrical conductively (σ) = 1.9×10^7 S/m

Relaxation time (τ) =?

Electrical conductivity of an electron is

$$\sigma = \frac{ne^2\tau}{m}$$

$$\text{or, } \tau = \frac{m \times \sigma}{ne^2}$$

$$= \frac{9.1 \times 10^{-31} \times 1.9 \times 10^7}{2.5 \times 10^{22} \times 1.6^2 \times 10^{-38}} = 2.7 \times 10^{-8} \text{ sec}$$

12. **Derive the relation for magnetic field on the axis of a circular loop and show that a circular current carrying coil behaves as a magnetic dipole for a larger distance.**

⇒ *Refer to Q.N. 12 of 070 Ashad*

13. **What radius is needed in proton synchrotron to attain particle energy of 15 GeV, assuming that a guide field of 2.0 Wb/m^3 is available? Rest mass of proton is 1.007529 Amu.**

⇒ Energy of particle = 15 GeV

Magnetic field (B) = 2.0 Wb/m^3

Mass (m) = 1.00 w7529 Amu

Rest mass of proton (m_0) = 1.007529 Amu

Radius (r) =?

Energy corresponding to m_0 is

$$m_0c^2 = m_0 \times 931 \text{ MeV}$$

$$= 1.007529 \times 0.931 \text{ GeV}$$

$$= 0.938 \text{ GeV}$$

Hence, total energy required = $E + m_0c^2$

$$= (15 + 0.938) \text{ GeV}$$

$$= 15.938 \text{ GeV}$$

This is the equivalent mass of proton.

Hence, radius (r) = $\frac{mv}{Be}$

$$= \frac{15.938 \times 1.66 \times 10^{-27} \times 3 \times 10^8}{2 \times 1.6 \times 10^{-19}}$$

$$= 17.5 \text{ m}$$

OR,

A parallel plate capacitor with circular plates is being charged by time varying electric field of 1.5 × 10^{12} V/ms. Calculate the induced magnetic field if the radius of the plate is 55mm and displacement current of the system.

⇒ Radius of the plate (r) = 55 mm = 55×10^{-3}m

$$\frac{dE}{dt} = 1.5 \times 10^{12} \text{ V/ms}$$

Displacement current is

$$i_d = \varepsilon_0 A \frac{dE}{dt}$$

...ynchrotron to attain
...g that a guide field of
...of proton is 1.007529

...Amu

m_0c^2
+ 0.938) GeV
...938 GeV
...ton.

$\frac{...10^{-27}\times 3\times 10^{8}}{...10^{-19}}$

...ith circular plates is being ...tric field of 1.5×10^{12} V/ms. ...tic field if the radius of the ...ent current of the system.

...= 55×10^{-3} m

Displacement current density is

$$j_d = \frac{i_d}{A} = \varepsilon_0 \frac{dE}{dt} = 8.85 \times 10^{-12} \times 1.5 \times 10^{12} = 13.275 \text{ A/m}^2$$

Induced magnetic field is

$$B = \frac{1}{2}\mu_0 r j_d$$

$$= \frac{1}{2} \times 4\pi \times 10^{-7} \times 55 \times 10^{-3} \times 13.275 = 4.58 \times 10^{-7} \text{ Tesla}$$

14. **Derive the relation for rise and fall of current in LR circuit. Plot a graph between current and time and explain the figure.**

⇒ *Refer to Q. N. 13 of 2069 Bhadra*

15. **Write Maxwell's electromagnetic wave equations in dielectric medium. Obtain electromagnetic wave equations for $\vec{E}$ and $\vec{B}$ in both dielectric medium and in free space. Compare velocity of electromagnetic wave in dielectric medium to free space.**

⇒ *Refer to Q.N. 15 of 069 Poush*

16. **Calculate the permitted energy levels of an electron in one dimensional potential well of width 0.2nm.**

⇒ Width of potential well (l) = 0.2 nm = 2×10^{-10} m

Permitted energy level of an electron in one dimensional potential well is given by

$$E_n = \frac{\pi^2 \hbar^2}{2m\,l^2} n^2$$

$$= \frac{\pi^2 (1.05 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} n^2 = 1.5 \times 10^{-18} n^2 \text{J}$$

$$\therefore E_n = \frac{1.5 \times 10^{-18}}{1.6 \times 10^{-19}} n^2 \text{ eV} = 9.38 n^2 \text{ eV}$$

Minimum energy that electron have is $E_1 = 9.38$ eV corresponding to n = 1.

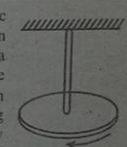
Other values of energy are $E_2 = 4E_1 = 37.5$eV.

$E_3 = 9E_1 = 84.3$eV, and so on.

1. **Differentiate between linear and angular harmonic motion. Show that the motion of torsion pendulum is angular harmonic motion. Also, find its time period.**

⇒

Linear harmonic motion	Angular harmonic motion
1. Here, acceleration produced on particle at any point is directly proportional to the linear displacement.	1. Here, acceleration produced on particle at any point is directly proportional to the angular displacement.
2. Restoring force is provided by weight, force constant. Examples: simple pendulum, bar pendulum, mass-spring system, etc.	2. Restoring torque is provided by torsional constant. Example: torsional pendulum



Torsion pendulum consist of a disc suspended at the end of a wire, as shown in figure other end of the wire is fixed at a rigid support when the disc is rotated the wire is twisted by an angle θ. Then restoring torque is created on it obeying Hook's law. The restoring torque is directly proportional to the angular displacement of wire.

$$\tau \propto \theta$$

$$\tau = -C\theta$$

where C is called couple per unit twist or torsional constant and is given by $C = \frac{\pi\eta r^4}{2l}$, r is radius and l is length of the wire.

Rotational form of Newton's 2nd law is

$$\tau = I\alpha$$

4m × 5m = 120m³

average distance covered by a
air between two consecutive
lls of the room.

c

2(l b + bh + l h)
2(6 × 4 + 4 × 5 + 6 × 5)
148m²

= 3.242 m

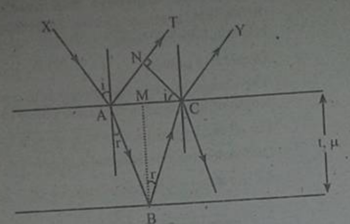
s made per second by sound wave

how that interference in thin film
and transmitted lights are

distribution of energy due to
light waves is called interference of

ilm due to reflected light

f thickness t and refractive index μ. A
rtially reflected along AT and partially
At B, light is reflected along BC and
Y.



Path difference between the rays AT and CY is

$$x = \mu (AB + BC) - AN \text{ ......(1)}$$

In ΔABM, $\cos r = \frac{BM}{AB} \Rightarrow AB = \frac{t}{\cos r}$

Similarly, $BC = \frac{t}{\cos r}$ .....(2)

$\therefore AB + BC = \frac{2t}{\cos r}$ ....(3)

Again in Δ ANC, $\sin i = \frac{AN}{AC} \Rightarrow AN = AC \sin i$

or, $AN = (AM + MC) \sin i$ ....(4)

In Δ ABM, $\tan r = \frac{AM}{MB} = \frac{AM}{t}$

or, $AM = t \tan r$ ....(5)

$\therefore AN = (AM + MC) \sin i = 2t \tan r \sin i$

Now, equation (i) becomes

$$x = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$$

$$= \frac{2t}{\cos r}\left[\mu - \sin r . \frac{\sin i}{\sin r} \times \sin r\right]$$

$$= \frac{2\mu t}{\cos r}[1 - \sin^2 r] \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$\therefore x = 2\mu t \cos r$ ....(6)

Since the reflection takes place from denser to rarer medium, according to electromagnetic wave theory further phase difference of π equivalent of path difference $\frac{\lambda}{2}$ occurs.

$\therefore$ Path difference is $x = 2\mu t \cos r - \frac{\lambda}{2}$

Case I:

If path difference is an integer multiple of λ, bright fringes will be observed.

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$\therefore 2\mu t \, \text{Cos}\, r = (2n + 1)\frac{\lambda}{2}$ ....(7)

Case II:

If the path difference is odd integer multiple of half wave length, dark fringes will be observed.

$$2\mu t \cos r - \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$\therefore 2\mu t \cos r = n\lambda$ ....(8)

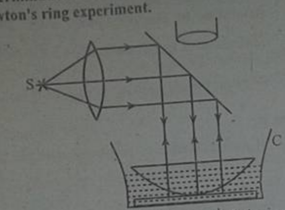
Note that for transmitted light, equation (7) holds for dark fringes and equation (8) holds for bright fringes. So the interference due to transmitted light and reflected light is complementary.

OR,

What are Newton's rings? How can you determine the refractive index of given liquid using Newton's rings experiment?

⇒ Circular interference fringes are produced by enclosing thin air film of varying thickness between plane glass plate and plano convex lens of large radius of curvature, such rings are called Newton's rings. For monochromatic light, dark and bright fringes are produced whereas for white light, coloured fringes are observed.

Determination of refractive index of a given liquid using Newton's ring experiment.



First of all, diameter of n^{th} and m^{th} dark ring is measured with the help of travelling microscope without liquid.

For air, $D_n^2 = 4n\lambda R$ ....(1)

$$D_m^2 = 4m\lambda R \text{ ....(2)}$$

The liquid is poured in the container C without disturbing the arrangement and again, diameters of n^{th} and m^{th} dark ring are measured which is found to be

$$D_n'^2 = \frac{4n\lambda R}{\mu} \text{ ....(3)}$$

$$D_m'^2 = \frac{4m\lambda R}{\mu} \text{ ....(4)}$$

Now, from equation (1) and (2)

$$D_m^2 - D_n^2 = 4(m-n)\lambda R \text{ ....(5)}$$

From equation (3) and (4), we get

$$D_m'^2 - D_n'^2 = \frac{4(m-n)\lambda R}{\mu} \text{ ....(6)}$$

Dividing equation (5) by (6), we get

$$\frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2} = \mu$$

This gives the value of refractive index of liquid.

5. **Explain the dispersive and resolving power of a diffraction grating. Derive expressions and develop a relation between them.**

⇒ **Dispersive power**

The change in angle of diffraction to a unit change in wave length of light used is dispersive power of the grating.

For n^{th} order maxima, $(a + b)\, \text{Sin}\theta = n\lambda$ ....(i)

Differentiating this equation will give

$$(a + b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$\boxed{\therefore \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

For small angle θ, $\cos\theta$ is practically constant so that $d\theta$ is directly proportional to $d\lambda$.

Resolving power of grating is its ability to show two neighboring lines in a spectrum as separate.

Two lines of wavelength λ and $\lambda + d\lambda$ are said to be just resolved if the central maxima due to $\lambda + d\lambda$ falls on first minima of λ. The resolving power of grating is given by $\frac{\lambda}{d\lambda}$

Consider two wavelets of wavelengths λ and $\lambda + d\lambda$ to be incident normally on surface of grating. The two wavelengths will give their own diffraction patterns. According to Rayleigh's criteria, the two patterns would be just resolved if the principal maxima of one falls on the first secondary minima of the other in any order.

The condition for the first secondary minima after the n^{th} order maxima is given by

$$(a + b)\sin(\theta + d\theta) = n\lambda + \frac{\lambda}{N} \text{ ....(1)}$$

Let n^{th} order principal maxima corresponding to wavelength $\lambda + d\lambda$ has same direction as that of the first secondary minimum of the n^{th} order principal maximum corresponding to wavelength λ.

Then, $(a + b) \sin(\theta + d\theta) = n(\lambda + d\lambda)$ ....(2)

From (1) & (2),

$$n\lambda + \frac{\lambda}{N} = n\lambda + n\, d\lambda$$

$$\boxed{\therefore \frac{\lambda}{d\lambda} = Nn}$$

This is the required equation for resolving power.

$$\text{Dispersive power} \left(\frac{d\theta}{d\lambda}\right) = \frac{n}{(a + b)\cos\theta}$$

$$= \frac{nN}{\cos\theta}$$

i.e., $\boxed{\text{Dispersive power} = \frac{\text{Resolving power}}{\cos\theta}}$

6. **A 200mm long tube containing 48cm of sugar solution produces an optical rotation of 11° when placed on a sacchari meter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of solution.**

⇒ *Refer to Q.N. 8 of 068 Magh*

7. **Prove that the condition for achromatism for the combination of two lenses of focal length f_1 and f_2 having dispersive power ω_r and ω_2 placed at a separate distance x is $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)$.**

⇒ Let f_1, f_2, and x be the focal lengths of two lenses and their distance of separation respectively.

The focal length of equivalent lens is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \quad \text{........(1)}$$

Differentiating this equation, we have

$$d\left(\frac{1}{F}\right) = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} - x\left\{\frac{-df_1}{f_1^2 f_2} - \frac{df_2}{f_1 f_2^2}\right\}$$

For the combination to be achromatic, we must have

$$d\left(\frac{1}{F}\right) = 0$$

Also, $\omega_1 = -\frac{df_1}{f_1}$ and $-\frac{df_2}{f_2} = \omega_2$

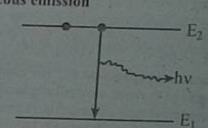
(which are dispersive power of material of lens.)

So, $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - x\left(\frac{\omega_1}{f_1 f_2} + \frac{\omega_2}{f_1 f_2}\right) = 0$

$\boxed{\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)}$ proved.

8. **Differentiate between spontaneous and stimulated emission of radiation. Explain the construction and working of He-Ne laser with a suitable energy level diagram.**

⇒ **Spontaneous emission**

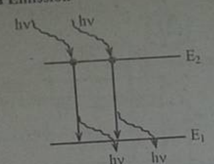


When an atom in excited state E_2 goes to the ground state E_1 by spontaneously emitting a photon, the process is known as spontaneous emission and the frequency of photon is given by

$$\upsilon = \frac{E_2 - E_1}{h}$$

In such a case, the emitted photon can move in any random direction. Photons emitted from various atoms in the assembly have no phase relationship between themselves and thus, the radiation given out is incoherent. Example: ordinary light.

Stimulated Emission



When an atom in an excited state E_2 interacts with an incident photon of frequency υ and is thereby induced to move to the ground state E_1 by emitting the difference of energy $E_2 - E_1$ as the photon of the same frequency, the process is known as stimulated emission. In such a case,

i. The emitted photon travels in the same direction in which the incident photon is moving.

ii. The two photons are in phase with each other.

For the He–Ne laser, refer to Q.N. 6 of 070 Ashad

9. **Derive an expression for the electric field at a point P at a distance R from a circular plastic disc of radius 'a' along its central axis. Does this expression for B reduce to an expected result for x>>a?**

⇒ *Refer to Q.N. 9 of 2070 Bhadra*

10. **A capacitor of capacitance 'C' is discharged through a resistor of resistance 'R'. After how many times constants is the energy stored becomes one fourth of initial value?**

⇒ Energy stored in the capacitor at any time is given by

$$U = \frac{q^2}{2C} = \frac{q_0^2\, e^{\frac{-2t}{RC}}}{2C} = U_0\, e^{\frac{-2t}{RC}} \qquad [\because q = q_0 e^{\frac{-2t}{RC}}]$$

where $U_0 = \frac{1}{2C} q_0^2$ is initial energy stored in the capacitor.

We have, $U = \frac{U_0}{4}$

$$\text{or, } \frac{U_0}{4} = U_0\, e^{\frac{-2t}{RC}}$$

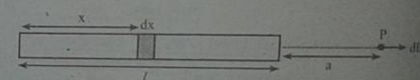
$$\text{or, } \ln 4 = \frac{2t}{RC}$$

$$t = \frac{RC \ln 4}{2} = \frac{\tau_c \ln 4}{2}; \quad \tau_c = RC \text{ is capacitive time constant.}$$

$$\boxed{\therefore t = 0.693\, \tau_c}$$

11. **Calculate the electric field due to a uniformly charged rod of length 1 at a point along its long axis at a distance 'a' from its nearest end.**

⇒



Let the rod be lying along x-axis and has uniform charge per unit length, λ. The rod can be divided into elementary segments of length dx and electric field intensity due to this segment which is at a distance x from far end of the rod is

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 (l + a - x)^2}$$

Total field due to whole rod is

$$E = \int_0^l \frac{\lambda dx}{4\pi\varepsilon_0 (l + a - x)^2} = \frac{\lambda}{4\pi\varepsilon_0} \int_0^l \frac{dx}{(l + a - x)^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\frac{1}{(l + a - x)} \right]_l^0 = \frac{\lambda l}{4\pi\varepsilon_0 a(l + a)}$$

$$\therefore E = \frac{q}{4\pi\varepsilon_0\, a(l + a)}$$

12. **Explain the principle and working of cyclotron. Show that the time spent by the particle in a Dee is independent of its speed and radius of its circular path.**

⇒ *Refer Q.N. 14 of 069 Chaitra*

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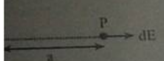
axis at a distance



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$\frac{dx}{(l + a - x)^2}$

$(l + a)$

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e particle in a Dee is
dius of its circular path.

OR,

Use Biot-Savart's law to calculate magnetic field on the axial line of a current carrying circular loop. Explain how the coil behaves for a large distance point.

⇒ *Refer to Q.N. 12 of 070 Bhadra*

13. **A copper strip 150 μm thick is placed in a magnetic field of strength 0.65 T perpendicular to the plane of the strip and current of 23 A is set up in the strip. Calculate: (i) the Hall voltage (ii) Hall coefficient and (iii) Hall mobility, if the number of electrons per unit volume is 8.5 × 10^{28}/m^3 and resistivity is 1.72 × 10^{-8} Ohm-m.**

⇒ Thickness (t) = 150 μm = 150 × 10^{-6} m

Magnetic field (B) = 0.65 T

Current (i) = 23 A

Number density (n) = $8.5\times10^{28}/m^3$

Resistivity (ρ) = $1.72\times10^{-8}\ \Omega-m$

(i) $V_H = \frac{Bi}{net} = \frac{0.65 \times 23}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 150 \times 10^{-6}}$

$= 7.33 \times 10^{-6}$ V

(ii) Hall coefficient $(R_H) = \frac{1}{ne} = \frac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$

$= 0.735 \times 10^{-10}\ m^3/coulomb$

(iii) Hall mobility (μ) = $R_H\, \sigma = \frac{R_H}{\rho}$

$= \frac{0.735 \times 10^{-10}}{1.72 \times 10^{-8}}$

$= 4.27 \times 10^{-3}\ m^2/V\ sec$

14. **A parallel plate capacitor with circular plates of 10 cm radius is charged producing uniform displacement current of magnitude 20A. Calculate (i) dE/dt in the region (ii) Displacement current density and (iii) Induced magnetic field.**

⇒ Radius of plate (r) = 10 cm = 0.1 m

Displacement current (i_d) = 20 A

$$i_d = \varepsilon_0 A \frac{dE}{dt}$$

$$\text{or, } \frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{20}{8.85\times10^{-12}\times\pi\,(0.1)^2} = 71.9\times10^{12}\ V/ms$$

i. Displacement current density is

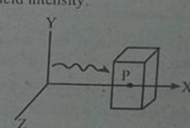
$$j_d = \varepsilon_0 \frac{dE}{dt} = 8.85\times 10^{-12} \times 71.9\times10^{12} = 636.62\ A/m^2$$

iii. Induced magnetic field is

$$B = \frac{1}{2}\mu_0\, r\, j_d = \frac{1}{2} 4\pi\times10^{-7}\times0.1\times636.62 = 4\times 10^{-5}\ \text{Tesla}$$

15. **Obtain an expression for energy transfer rate by electromagnetic wave. From your result, show that I α E^2_{rms}. Where I is the intensity of EM wave and E_{rms} is root mean square value of electric field.**

⇒ The rate of flow of energy in an electromagnetic wave per unit area is described by a vector $\vec{S}$ called Poynting vector, which is the cross product of electric field intensity and magnetic field intensity.



Consider an electromagnetic wave propagating along positive x–axis. The total energy at any instant dt stored in a box while travelling along within element dx and face area A is

$$dU = dU_E + dU_B = (u_E + u_E)\, dv = (u_E + u_B)\, A\, dx$$

where u_B and u_E are magnetic and electric energy densities

For free space, $u_E = u_B$

$$\text{or, } dU = 2u_E\, Adx = 2\left(\frac{1}{2}\varepsilon_0 E^2\right) A\, dx$$

$$\frac{dU}{A} = \varepsilon_0 E^2 dx \quad [\because c = \frac{E}{B} \text{ and } c = \frac{dx}{dt}]$$

$$= \varepsilon_0 E (cB)\, c\, dt$$

$$= \varepsilon_0 EBc^2 dt,$$

where c is velocity of light and is expressed as $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

or, $\frac{dU}{Adt} = \frac{EB}{\mu_0}$

$$\boxed{\therefore S = \frac{EB}{\mu_0}}$$

The average value of $\vec{S}$ gives intensity of electromagnetic wave

$$I = S_{avg} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c}$$

or, $I = \frac{E_{rms}^2}{\mu_0 c}$ $\quad \left[\because E_{rms}^2 = \frac{E_0^2}{2}\right]$

Hence, $\boxed{I \propto E_{rms}^2}$ proved.

16. **Derive the Schrodinger time independent wave equation. Also what do you mean by a potential barrier?**

⇒ **Schrodinger time independent wave equation**

The wave function of a particle in one dimension is

$$\Psi(x, t) = Ae^{-\frac{i}{\hbar}(Et - px)} \quad \text{.............(i)}$$

Differentiating equation (i) with respect to x.

$$\frac{d\Psi}{dx} = \frac{ip}{\hbar} Ae^{-\frac{i}{\hbar}(Et - px)} \quad \text{.............................(ii)}$$

Again differentiating with respect to x.

$$\frac{d^2\Psi}{dx^2} = \left(\frac{ip}{\hbar}\right)^2 Ae^{-\frac{i}{\hbar}(Et - px)} = \frac{-p^2}{\hbar^2}\Psi \quad \text{.............(iii)}$$

Total energy of a particle is

$$E = K + V = \frac{1}{2}mv^2 + V = \frac{p^2}{2m} + V$$

Multiplying both sides by Ψ and rearranging, we get

$$p^2\Psi = 2m(E - V)\Psi \quad \text{..........(iv)}$$

Then, equation (iii) becomes

$$\frac{d^2\Psi}{dx^2} = \frac{-2m(E - V)\Psi}{\hbar^2}$$

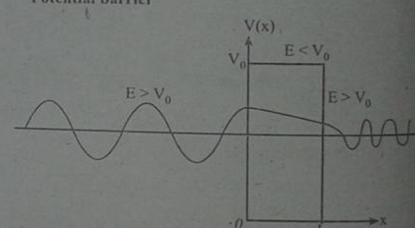
$$\therefore \frac{d^2\Psi}{dx^2} + \frac{2m(E - V)\Psi}{\hbar^2} = 0 \quad \text{...........(v)}$$

This is the time independent Schrodinger wave equation in one dimension.

Three dimensional time independent Schrodinger wave equation is

$$\nabla^2\Psi + \frac{2m(E - V)}{\hbar^2}\Psi = 0$$

Potential barrier



Let a finite potential V(x) energy function of height V_0 in the region of width L be considered. The downward facing region of the potential function is called potential barrier. For a particle, it is barrier because a particle should have minimum energy of V_0 in order to reach the other side. However, if the barrier is relatively narrow, the particle can tunnel through to cross the barrier.

1. **Obtain an expression for the time period of a compound pendulum and show that its time period is unaffected by the fixing of a small additional mass to it at its centre of suspension.**

⇒ Consider a rigid body of any shape and mass m capable of oscillating freely about horizontal axis passing through it and perpendicular to its plane.

Let O be centre of suspension of the body and G its centre of gravity vertically below at distance l in position of rest. When a body is displaced through small angle θ, centre of gravity is shifted to G' and its weight mg acts vertically downward at G'.

If the pendulum is now released, a restoring couple acts on it and brings it back to the initial position.

Restoring torque is

$$\tau = -mg \times G'A$$

$$= -mgl\sin\theta$$

$$= -mgl\theta \quad \text{[since for small } \theta, \sin\theta \approx \theta]$$

This restoring couple gives rise to an angular acceleration α in the pendulum. If I is moment of inertia of rigid body about an axis passing through its centre of suspension, then restoring couple is given by

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\text{or, } I\frac{d^2\theta}{dt^2} = -mgl\theta \quad \text{i.e., } \frac{d^2\theta}{dt^2} \propto \theta$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0$$

Time period of compound pendulum is $T = 2\pi\sqrt{\frac{I}{mgl}}$

The moment of inertia of the pendulum about an axis passing through O and perpendicular to its plane is $mk^2 + ml^2$, k being the radius of gyration about an axis passing through centre of gravity.

$$\therefore \frac{d^2\theta}{dt^2} + \frac{mgl}{mk^2 + ml^2}\theta = 0$$

Comparing it with differential equation of S.H.M., we get the time period as

$$T = 2\pi\sqrt{\frac{k^2 + l^2}{g\,l}}$$

or, $T = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$ which is independent of mass.

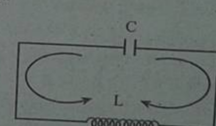
If small mass is attached at centre of suspension, then there will be no change in moment of inertia of the rigid body (compound pendulum).

OR,

What is electromagnetic oscillation? Derive differential equation of damped LCR oscillation and find its frequency.

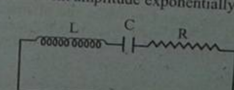
⇒ **Electromagnetic oscillation**

The circuit containing both a capacitor C and an inductor L forms an electromagnetic oscillator in which current varies sinusiodally with time resulting an electromagnetic oscillation.



Damped LCR oscillation

Fig. shows LCR circuit. In damped oscillation, amplitude of oscillation decreases exponentially with time due to external resistive force. Here, R stands as resistive factor which decays the current amplitude exponentially with time.



By the principle of conservation of energy, we have

$$U = U_B + U_E = \frac{1}{2} Li^2 + \frac{q^2}{2C} \quad \text{.........(1)}$$

Since rate of change of energy is power, we write

$$\text{i.e., } \frac{dU}{dt} = -i^2R \quad \text{.........(2)}$$

where –ve sign represents that the energy stored, U decreases with time.

$$\text{From (1), } \frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} \quad \text{.......(3)}$$

$$\text{From (2) \& (3), } Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R$$

$$\text{or, } Li\frac{d^2q}{dt^2} + \frac{q}{C}i + i^2R = 0 \quad \left[\because i = \frac{dq}{dt}\right]$$

$$\left(L\frac{d^2q}{dt^2} + \frac{q}{C} + iR\right) = 0$$

$$\boxed{\therefore L\frac{d^2q}{dt^2} + \frac{dq}{dt}R + \frac{q}{C} = 0}$$

This is the required differential equation of damped LCR which can be compared with mechanical damped oscillation.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

This will give

$$m = L,\ b = R,\ K = \frac{1}{C}$$

$$\text{Angular frequency } (\omega) = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\text{Frequency (f)} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

2. **A particle is moving with simple harmonic motion in a straight line. If it has a speed v_1 when the displacement is x_1 and speed v_2 when the displacement is x_2 then, show that the amplitude of motion is $a = \left(\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right)^{\frac{1}{2}}$.**

⇒ Let the motion of particle be represented by

$$x = a \sin\omega t \quad \text{..................(i)}$$

$$\text{Its velocity is } v = \frac{dx}{dt} = a\,\omega\cos\omega t \quad \text{......................(ii)}$$

$$= \omega\sqrt{a^2 - x^2}$$

For displacement x_1 and velocity v_1,

$$x_1 = a \sin\omega t \text{ and } v_1 = \omega\sqrt{a^2 - x_1^2} \quad \text{..................(iii)}$$

For displacement x_2 and velocity v_2,

$$v_2 = \omega\sqrt{a^2 - x_2^2} \quad \text{.......................(iv)}$$

Dividing equation (iv) by (iii), we get

$$\frac{v_2}{v_1} = \frac{\sqrt{a^2 - x_2^2}}{\sqrt{a^2 - x_1^2}}$$

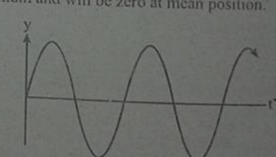
$$\text{or, } v_2^2 a^2 - v_2^2 x_1^2 = v_1^2 a^2 - v_1^2 x_2^2$$

$$\text{or, } a^2(v_2^2 - v_1^2) = v_2^2 x_1^2 - v_1^2 x_2^2$$

$$\therefore a = \left(\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right)^{\frac{1}{2}} \text{ proved.}$$

3. **In the progressive wave, show that the potential energy and kinetic energy of every particle will change with time but the average K.E. per unit volume and P.E. per unit volume remains constant.**

⇒ For a particle executing SHM, the velocity is maximum at equilibrium position and will be zero at extreme position. Since kinetic energy is proportional to square of velocity, it is maximum at mean position and zero at extreme position. Consequently, the potential energy at extreme position is maximum and will be zero at mean position.



To move particle from mean position to distance y, work has to be done against acceleration. For small displacement dy, work done is

$$dW = F\,dy$$

If ρ is density of the medium, work done per unit volume for small displacement dy is

$$dw = m\,a_p dy$$

$$= \rho a_p dy \quad ;a_p \text{ is acceleration, } m = \rho \text{ for unit volume}$$

$$= \rho\left[\frac{4\pi^2 a v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x)\right]dy$$

$$\text{Total work done (w)} = \int dw = \frac{4\pi^2 \rho v^2}{\lambda^2}\int_0^y a\sin\frac{2\pi}{\lambda}(vt - x)\,dy$$

Work done per unit volume is stored as potential energy per unit volume u which is given by

$$u = \frac{4\pi^2 \rho v^2}{\lambda^2}\int_0^y y\,dy$$

$$\text{or, } u = \frac{4\pi^2 \rho v^2}{\lambda^2}\frac{y^2}{2}$$

$$\text{or, } u = \frac{4\pi^2 \rho v^2}{\lambda^2}\frac{a^2}{2}\sin^2\frac{2\pi}{\lambda}(vt - x)$$

$$\therefore u = \frac{2\pi^2 \rho a^2 v^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(vt - x) \;\;.........\;(1)$$

Kinetic energy per unit volume is

$$k = \frac{1}{2}\rho\left(\frac{dy}{dt}\right)^2 \;;\; \rho = m \text{ for unit volume.}$$

$$= \frac{1}{2}\rho\left[\frac{2\pi a v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)\right]^2$$

$$\therefore k = \frac{2\pi^2 \rho a^2 v^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(vt - x) \;\;.............\;(2)$$

Equations (1) and (2) illustrate that the potential energy and kinetic energy of every particle will change with time.

Total energy (E) = k + u

$$= \frac{2\pi^2 \rho a^2 v^2}{\lambda^2}\left[\sin^2\frac{2\pi}{\lambda}(vt - x) + \cos^2\frac{2\pi}{\lambda}(vt - x)\right]$$

$$= \frac{2\pi^2 \rho a^2 v^2}{\lambda^2}$$

$$= 2\pi^2 \rho a^2 f^2 \qquad [\text{since } v = \lambda f]$$

Calculate average K.E and P.E per unit volume on your own

This shows that the average kinetic energy per unit volume and P.E. per unit volume are equal and each is equal to half the total energy per unit volume.

4. **Two coherent sources having constant phase but different amplitudes A_1 and A_2 superimpose, prove that the intensity of superimposed beam is $I = A_1^2 + A_2^2 + 2A_1 A_2\cos\delta$.**

⇒ Consider two waves having amplitudes A_1 and A_2 with phase difference δ.

$$\therefore \; y_1 = A_1\sin\omega t, \;\; y_2 = A_2\sin(\omega t + \delta)$$

Superimposed wave is $y = y_1 + y_2$

$$= A_1\sin\omega t + A_2(\sin\omega t + \delta)$$

$$= A_1\sin\omega t + A_2\sin\omega t\cos\delta + A_2\cos\omega t\sin\delta$$

$$\text{or, } y = (A_1 + A_2\cos\delta)\sin\omega t + A_2\sin\delta\cos\omega t.$$

Let $A_1 + A_2\cos\delta = R\cos\theta$ ....(1)

$A_2\sin\delta = R\sin\theta$ ....(2)

Then, $y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$

$= R\sin(\omega t + \theta)$; R is amplitude of resultant wave.

Squaring and adding (1) and (2), we get

$$(A_1 + A_2\cos\delta)^2 + A_2^2\sin^2\delta = R^2$$

$$A_1^2 + A_2^2\cos^2\delta + 2A_1A_2\cos\delta + A_2^2\sin^2\delta = R^2$$

$$\therefore R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

Since Intensity (I) ~ (amplitude)^2 ~ R^2

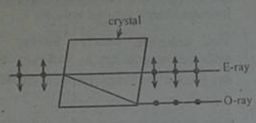
Hence, $I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$ proved.

OR,

Explain the phenomenon of double refraction. Describe the construction and action of Nicol prism.

⇒ **Double refraction:**

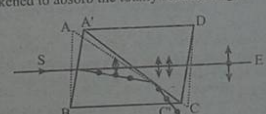
When an ordinary unpolarized light is incident on calcite or quartz crystal, the crystal splits the refracted rays into ordinary ray and extra-ordinary ray. This phenomenon is called double refraction.



Nicol prism: It is an optical device made from calcite crystal and used in many instruments for producing and analyzing plane polarized light.

Construction:

A calcite crystal about 3 times as long as it is wide is taken. The principal section ABCD is shown in Fig. Its end faces are cut down so as to reduce the angles at B and D from 71° to 68° with principal section. The crystal is then cut apart along A'C' perpendicular both to the principal plane and the end faces such that A'C' makes an angle of 90° with the ends A'B and C'D. The two cut surfaces are ground, polished optically flat, and cemented together with Canada balsam which is clear transparent cement whose refractive index lies mid-way between the refractive indices of calcite for the ordinary and the extra-ordinary rays. The sides of prism are blackened to absorb the totally reflected light.



Working of Nicol prism: [μ_o < cd < μ_e]

When a ray of light is incident on Nicol prism, it splits the ray into extra-ordinary ray and ordinary ray due to double refraction of the crystal. For ordinary ray, Canada balsam works as rare medium; so it suffers total internal reflection. For extra-ordinary ray, Canada balsam works as dense medium; so it does not escape but from prism and only E-ray is observed after refraction.

5. **White light is incident on a soap film at an angle $\sin^{-1}\left(\frac{4}{5}\right)$ and the reflected light on examination by a spectrometer shows dark bands. The consecutive dark bands correspond to wavelength 6.1×10^{-5} cm and 6.0×10^{-5} cm. If μ=1.33 for the film, calculate its thickness.**

⇒ $\lambda_1 = 6.0\times10^{-5}$ cm , $\lambda_2 = 6.1\times10^{-5}$ cm

Angle of incidence (i) = $\sin^{-1}\left(\frac{4}{5}\right)$

Refractive index (μ) = 1.33

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

$$\text{or, } \sin r = \frac{\sin \sin^{-1}\frac{4}{5}}{\mu} = \frac{4}{5\mu}$$

$$\text{or, } r = \sin^{-1}\left(\frac{4}{5\times1.33}\right) = 36.97°$$

For dark band due to reflected light,

$2\mu t \cos r = n\lambda_1$ ....(1)

$2\mu t \cos r = (n+1)\lambda_2$ ....(2)

From (1) & (2),

$n\lambda_1 = (n+1)\lambda_2$

or, $6.1\times10^{-5}\, n = (n+1)\, 6\times10^{-5}$

∴ $n = 60$

Using equation (1), we have

$2\mu t \cos r = n\lambda_1$

prism, it splits the
ray due to double
ray. Canada balsam
internal reflection
works as dense
prism and only E–

at an angle $\sin^{-1}\left(\frac{4}{5}\right)$.

**by a spectrometer
utive dark bands
cm and 6.0×10^{-5} cm.
ickness.**

or, $t = \frac{60\times6.1\times10^{-5}}{2\times1.33\times\cos 36.97^\circ}$

$\therefore$ $t = 1.72\times10^{-5}$ cm

6. **Light of wavelength 600 nm is incident normally on a slit of width 0.1 mm. Calculate the intensity at $\theta = 0.2^\circ$.**

⇒ Wavelength of light (λ) = 600 nm = 600×10^{-9} m

Slit width (d) = 0.1 mm = 10^{-4} m

Angle of diffraction (θ) = 0.2°

Intensity (I) = ?

We have,

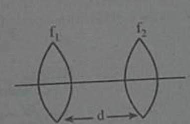
$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Here, $\alpha = \frac{\pi}{\lambda} d \sin\theta = \frac{\pi\times10^{-4}\sin 0.2}{6\times10^{-7}} = 1.82$

So, $I = I_0\left(\frac{\sin 1.82}{1.82}\right)^2 = 0.003\ I_0$ i.e., 0.3% of I_0

7. **Two lenses of focal lengths 8 cm and 4 cm are placed at a certain distance apart. Calculate the position of principal points if they form an achromatic combination.**

⇒



Here, $f_1 = 8$ cm, $f_2 = 4$cm

For achromatic condition,

$$d = \frac{f_1+f_2}{2} = \frac{12}{2} = 6\text{ cm}$$

Position of principal points are

$$\alpha = \frac{f.d}{f_2} \text{ and } \beta = -\frac{f.d}{f_1}$$

where focal length (f) of equivalent lens is calculated as

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{8\times4}{8+4-6} = \frac{32}{6} = 5.3\text{ cm}$$

$$\therefore \alpha = \frac{5.3\times6}{4} = 7.95\text{ cm}$$

$$\beta = \frac{-5.3\times6}{8} = -3.97\text{ cm}$$

8. **An optical fiber has a NA of 0.2 and a cladding refractive index of 1.59. Determine acceptance angle for the fiber in water which has a refractive index of 1.33.**

⇒ Numerical aperture (NA) = 0.2

Cladding refractive index (n_2) = 1.59

Acceptance angle (θ) = ?

Refractive index (μ) = 1.33

We have, $NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$ .... (i)

When fibre is in air, $n_0 = 1$ and $NA = \sqrt{(n_1)^2-(n_2)^2} = 0.2$

or, $n_1 = \sqrt{(0.2)^2+1.59^2} = 1.6025$

When the fibre is in water, $n_0 = 1.33$

$$NA = \sqrt{\frac{n_1^2-n_2^2}{n_0}} = \sqrt{\frac{1.6025^2-1.59^2}{1.33}} = 0.15$$

Acceptance angle $\theta_0(\max) = \sin^{-1}(NA) = \sin^{-1}(0.15) = 8.6^\circ$

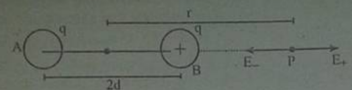
9. **A ring has a charge q uniformly distributed in it. Find the expression for the electric field at any point on the axial line of the ring. Locate the point at which the field is maximum.**

⇒ *Refer to Q.N. 9 of 070 Ashad*

OR,

Prove that electric field due to a short dipole at axial point is twice that at equatorial point.

Field along axial line of dipole



Field along axial line of dipole

Consider a dipole AB separated by distance 2d, field at P at distance r from centre of dipole due to +q and −q charge is to be measured.

Here, Resultant field is $E = E_+ + E_-$

$$E = \frac{q}{4\pi\varepsilon_0(r-d)^2} - \frac{q}{4\pi\varepsilon_0(r+d)^2}$$

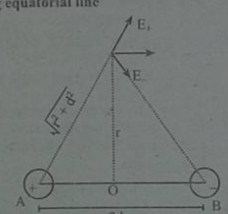
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4rd}{(r^2-d^2)^2}\right];\ P = 2d.q \text{ is dipole moment}$$

$$= \frac{2pr}{4\pi\varepsilon_0(r^2-d^2)^2}$$

For short dipole, r>>d

$$\therefore \vec{E}_{axial} = \frac{\vec{p}}{2\pi\varepsilon_0 r^3} \text{......(1)}$$

Field along equatorial line



+q & −q charges set up electric field E_+ & E_- respectively.

So, $|\vec{E}_+| = |\vec{E}_-|$

Total electric field at P is vector sum and is expressed as

$$|\vec{E}| = |\vec{E}_+|\cos\theta + |\vec{E}_-|\cos\theta$$

$$\therefore E = \frac{q\cos\theta}{4\pi\varepsilon_0(r^2+d^2)} + \frac{q\cos\theta}{4\pi\varepsilon_0(r^2+d^2)} \quad \left[\because \cos\theta = \frac{d}{\sqrt{r^2+d^2}}\right]$$

$$= \frac{2qd}{4\pi\varepsilon_0(r^2+d^2)^{\frac{3}{2}}}$$

$$= \frac{P}{4\pi\varepsilon_0(r^2+d^2)^{\frac{3}{2}}} \quad \text{[where } p = 2qd \text{ is dipole moment]}$$

For short dipole r>>d,

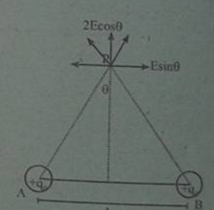
$$\vec{E}_{eq} = \frac{\vec{p}}{4\pi\varepsilon_0 r^3} \text{........(2)}$$

From equation (1) and (2), we conclude

$\boxed{\vec{E}_{axial} = 2\vec{E}_{eq}}$ proved.

10. **A particle of charge −q and a mass m is placed midway between two equal positive charge q_0 of separation d. If the negative charge −q is displaced in perpendicular direction to the line joining them and released, show that the particle describe a SHM with a period $T = \left[\frac{\varepsilon_0 m\pi^3 d^3}{qq_0}\right]^{\frac{1}{2}}$**

⇒



and is expressed as

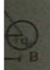
$[\because \cos\theta = \frac{d}{\sqrt{r^2+d^2}}]$

2qd is dipole moment]

de

s m is placed midway
$_0$ of separation d. If the
perpendicular direction
, show that the particle

$\frac{_0 m\pi^3 d^3}{qq_0}\Big]^{\frac{1}{2}}$



Let the particle of charge $-q$ and mass m be at perpendicular distance x from the line joining two charges $+q_0$

Electric field produced by q_0 at point P is

$$E = \frac{q_0}{4\pi\varepsilon_0(d^2/4 + x^2)} \text{ ....(i)}$$

This field can be resolved into two components $E\cos\theta$ along vertical direction and $E\sin\theta$ along horizontal direction, horizontal components cancel each other being equal and opposite.

So, resultant electric field along vertical direction due to both charge is

$2E\cos\theta$

$$= \frac{2q_0\cos\theta}{4\pi\varepsilon_0(d^2/4 + x^2)}$$

$$= \frac{2q_0}{4\pi\varepsilon_0(d^2/4 + x^2)} \times \frac{x}{\sqrt{\frac{d^2}{4} + x^2}}$$

$$= \frac{2q_0 x}{4\pi\varepsilon_0(d^2/4 + x^2)^{3/2}} \times$$

Force experienced by charge $-q$ at point p is

$$F_x = \frac{-2qq_0 x}{4\pi\varepsilon_0(d^2/4 + x^2)^{3/2}}$$

$$\text{or, } m\frac{d^2x}{dt^2} = \frac{-2qq_0 x}{4\pi\varepsilon_0(d^2/4 + x^2)^{3/2}}$$

$$\text{or, } \frac{d^2X}{dt^2} + \frac{2qq_0 x}{4m\pi\varepsilon_0(d^2/4 + x^2)^{3/2}} = 0$$

This is the differential equation of simple harmonic motion with angular frequency

$$\omega^2 = \frac{2qq_0}{4m\pi\varepsilon_0(d^2/4 + x^2)^{3/2}}$$

$$\text{Time period } (T) = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{4\pi\varepsilon_0 m(d^2/4 + x^2)^{3/2}}{2qq_0}}$$

At centre, $x = 0$

$$\therefore T = \left(\frac{\varepsilon m\pi^3 d^3}{qq_0}\right)^{1/2} \text{ proved.}$$

11. A cylindrical capacitor has radii a and b. Show that half the stored electric potential energy lies within a cylinder of radius $r = \sqrt{ab}$.

⇒ Capacitance of capacitor having internal radius a and external radius b is

$$C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)} \text{ ....(i)}$$

Capacitance of capacitor with inner radius a and outer radius $\sqrt{ab}$ is

$$C' = \frac{2\pi\varepsilon_0 l}{\ln(\sqrt{ab}/a)} = \frac{4\pi\varepsilon_0 l}{\ln(b/a)} \text{ ....(ii)}$$

Now, energy in first case is

$$E = \frac{q^2}{2C} = \frac{q^2}{2\times 2\pi\varepsilon_0 l}\ln(b/a) \text{ ....(iii)}$$

Energy in second case, $E' = \frac{q^2}{2\times 4\pi\varepsilon_0 l}\ln(b/a)$..........(iv)

Dividing (iv) by (iii), we get

$$\frac{E'}{E} = \frac{1}{2}$$

$$\therefore E' = \frac{1}{2}E$$

Hence, half the stored electric potential energy lies within a cylinder of radius $r = \sqrt{ab}$.

12. A flat silver strip of width 1.5 cm and thickness 1.5 mm carries a current of 150 A. A magnetic field of 2.0 Tesla is applied perpendicular to the flat face of the strip. The emf developed across the width of strip is measured to be

17.9 μV. Estimate the number density of free electrons in the metal.

⇒ Thickness (t) = 1.5 mm = 1.5×10^{-3} m

Width (b) = 1.5cm = 1.5×10^{-2} m

Current (I) = 150 A

Mag. field (B) = 2.0 T

No. density (n) = ?

Hall Emf (V_H) = 17.9μV = 17.9×10^{-6} V

~~Hall voltage (V_H) = $\frac{E}{b} = \frac{17.9\times10^{-6}}{1.5\times10^{-2}} = 11.93\times10^{-4}$ V~~

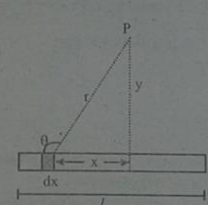
Also, $V_H = \frac{Bi}{net}$

$$\text{or, } n = \frac{Bi}{V_H et} = \frac{2\times150}{11.93\times10^{-4}\times1.6\times10^{-19}\times1.5\times10^{-3}}$$

$$= 64.76\times10^{26}/m^3$$

13. **A straight wire segment of length l carries current I. Show that the magnetic field b produced by the segment at a displace y from it along a perpendicular bisector is B $= \frac{\mu_0 I}{2\pi y}\frac{l}{\sqrt{(l^2+4y^2)}}$**

⇒



Magnitude of differential magnetic field produced at P due to elemental length dx located at distance r from P is

$$dB = \frac{\mu_0 I}{4\pi}\frac{dx \sin\theta}{r^2} \;\ldots\ldots(1)$$

The direction of $d\vec{B}$ in Fig. is that of vector $d\vec{l} \times \vec{r}$. Total magnetic field at P due to whole length is given as

$$B = 2\int_0^{l/2} dB$$

$$\text{or, } B = \frac{\mu_0 I}{2\pi}\int_0^{l/2}\frac{\sin\theta}{r^2}dx$$

From figure, $r = \sqrt{x^2+y^2}$, $\sin\theta = \frac{y}{\sqrt{x^2+y^2}} = \sin(\pi-\theta)$

$$B = \frac{\mu_0 I}{2\pi}\int_0^{l/2}\frac{y\,dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$\therefore B = \frac{\mu_0 I}{2\pi}\left[\frac{x}{(x^2+y^2)^{\frac{1}{2}}}\right]_0^{\frac{l}{2}} = \frac{\mu_0 I}{2\pi}\frac{l}{\sqrt{(l^2+4y^2)}}$$

14. **Find the inductance of a toroid having N number of turns and radius R.**

⇒ *Refer to Q.N. 14 of 069 Poush*

OR,

Show that the energy per unit volume in electric field and magnetic field are proportional to the square of their fields.

15. **State and explain Maxwell's equations. Derive the continuity equation: $\nabla.\vec{J} = -\frac{\partial\rho}{\partial t}$.**

⇒ *Refer to Q.N. 15 of 069 Chaitra*

16. **Determine the total energy of a particle using Schrodinger equation, when the potential energy has value V = 0 for 0 < x < a and V = a for x ≤ 0 and x ≥ a.**

⇒ Refer to QN. 16 of 2069 Poush

1. **What are drawbacks of simple pendulum? Show that the period of torsion oscillations remain unaffected even if the amplitude be large, provided that the elastic limit of the suspension wire is not exceeded.**

⇒ **Drawbacks of simple pendulum**

i. It is impossible to have both the point mass of the bob and weightless string.

ii. The resistance of air affect, the motion of bob.

iii. The relation $T = 2\pi\sqrt{\frac{L}{g}}$ is true only for oscillation of infinitely small amplitude.

iv. The motion of bob is not only linear but it has also rotatory motion which affects on time period of simple pendulum.

II part

Consider a torsional pendulum suspended at the end of the wire as shown in figure.

If θ be the angle turned by the wire then $C\theta$ will be restoring torsional couple which tends to bring the disc in its original position and given by

$\tau = -C\theta$

C is couple per unit twist and $C = \frac{\pi\eta r^4}{2l}$

r and l are radius and length of the wire respectively and η is modulus of rigidity of the wire.

Rotational form of Newton's second law is

$$\tau = I\alpha$$

$$\text{or, } \tau = I\frac{d^2\theta}{dt^2}$$

$$\text{or, } -C\theta = I\frac{d^2\theta}{dt^2}$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0 \quad \text{.........(1)}$$

Equation (1) shows that motion of torsional pendulum is simple harmonic in nature with angular frequency $\omega = \sqrt{\frac{C}{I}}$ and time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

This is the required expression for time period of torsional pendulum.Notice that in the derivation of this formula, no approximation is required as in simple pendulum. Hence, the period of torsional oscillation remains the same for large amplitude oscillation provided that the elastic limit of the suspension wire has not been exceeded.

OR,

In simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic energy and what fraction is potential energy? At what displacement is half kinetic energy and half potential energy?

⇒ Let simple harmonic motion of particle be represented by

$$y = A \sin \omega t \quad \text{........(1)}$$

Velocity of particle $\frac{dy}{dt} = A\,\omega \cos \omega t$ ....(2)

When displacement is half the amplitude

$$\frac{A}{2} = A \sin \omega t$$

$$\text{or, } \sin \omega t = \frac{1}{2}$$

and $\cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

We have, $K.E. = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2$

$$= \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$$

$$= \frac{1}{2} m \omega^2 A^2 \frac{3}{4} \text{ ....(1)}$$

Potential energy, $P.E. = \frac{1}{2} ky^2$

where k is force constant and $k = m\omega^2$

$$\text{or, } P.E. = \frac{1}{2} k A^2 \sin^2 \omega t$$

$$= \frac{1}{2} m \omega^2 A^2 . \frac{1}{4} \text{ ......(2)}$$

Total energy of the system is given by

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2 \text{ ....(3)}$$

$$\text{Now, } \frac{K..E.}{E} = \frac{\frac{3}{8} m \omega^2 A^2}{\frac{1}{2} m \omega^2 A^2} = \frac{3}{4}$$

$$\text{and, } \frac{P.E.}{E} = \frac{\frac{3}{8} m \omega^2 A^2}{\frac{1}{2} m \omega^2 A^2} = \frac{1}{4}$$

For K.E. to be half of total energy

$$\frac{\frac{3}{8} m \omega^2 A^2 \cos^2 \omega t}{\frac{1}{2} m \omega^2 A^2} = \frac{1}{2}$$

$$\text{or, } \cos^2 \omega t = \frac{1}{2}$$

and $\sin \omega t = \frac{1}{\sqrt{2}}$ $\left[\because \sin \omega t = \sqrt{1 - \cos^2 \omega t}\right]$

$$\therefore y = A \sin \omega t = \frac{A}{\sqrt{2}}$$

Hence, at displacement $\frac{A}{\sqrt{2}}$, particle has half kinetic energy and half potential energy.

2. **Derive differential equation of LC oscillation with the solution of this equation, show that the maximum value of electric and magnetic energies stored in LC circuit is equal.**

⇒ *Refer to Q. N. 2 of 069 Chaitra.*

3. **How much acoustic power enters the window of area 1.58 m^2, through the sound wave (standard intensity level 10^{-16} W/cm^2)? The window opens on a street where the street noise results in an intensity level at the window of 60 dB.**

⇒ Area of window (A) = 1.58 m^2

Standard intensity (I_0) = 10^{-16} W/cm^2 = 10^{-12} W/m^2

Intensity level = 60 dB

We know that, intensity level $= 10 \log\left(\frac{I}{I_0}\right)$

$$\text{or, } 60 = 10 \text{ kg} \frac{I}{10^{-12}}$$

$$\therefore I = 10^{-6} \text{ w/m}^2$$

Acoustic power = Intensity × Area

$$= 10^{-6} \times 1.58$$

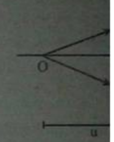
$$= 1.58 \times 10^{-6} \text{ watt}$$

4. **Explain circle of least confusion. Show that the diameter of a circle of least confusion is independent of the focal length of a lens.**

⇒ **Circle of least confusion:**

For a point object illuminated by white light and situated on the axis of the lens, coloured images are formed along the

axis. The bl...
the farthest...
placed at th...
aberration is...
confusion is...



Let u be the ...
red image. If...
then

Lens formula...

Taking v_b v_r = ...

But $f_r - f_b = \omega$...

In ∆s LQN an...

In ∆LPN and ...

particle has half kinetic energy

n of LC oscillation with the
how that the maximum value
nergies stored in LC circuit is

ra.

enters the window of area 1.58
vave (standard intensity level
v opens on a street where the
ntensity level at the window of

m^2

^{16} W/cm^2 = 10^{-12} W/m^2

el = 10 log $\left(\frac{I}{I_0}\right)$

0 kg $\frac{I}{10^{-12}}$

0^{-6} w/m^2

× Area

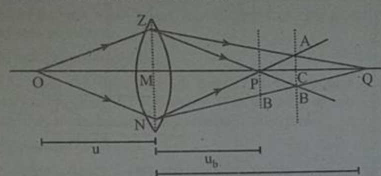
8

watt

nfusion. Show that the diameter
sion is independent of the focal

ted by white light and situated on
red images are formed along the

axis. The blue image is nearest the lens and the red image is the farthest. In between these two images, if a screen is placed at the position XY, the image of least chromatic aberration is formed. The diameter of the circle of least confusion is calculated as follow.



Let u be the distance of object $v_b < v_r$ be distance of blue & red image. If F_b and F_r are focal length for blue & red ray then

Lens formula: $\frac{1}{v_b} - \frac{1}{u} = \frac{1}{f_b}$ ......(1)

or, $\frac{1}{v_r} - \frac{1}{u} = \frac{1}{f_r}$

Taking $v_b\, v_r = v^2$ and $f_r . f_b = f^2$

$$\frac{v_r - v_b}{v^2} = \frac{f_r - f_b}{f^2}$$

But $f_r - f_b = \omega f$ .....(3)

$$\therefore \frac{v_r - v_b}{v^2} = \frac{\omega . f}{f^2} = \frac{\omega}{f}$$

$$v_r - v_b = \frac{\omega}{f} v^2$$

In Δs LQN and ΔAQB, $\frac{CQ}{AB} = \frac{MQ}{LN}$ ......(4)

In ΔLPN and ΔAPB, $\frac{PC}{AB} = \frac{MP}{LN}$ ....(5)

Adding (4) and (5): $\frac{CQ}{AB} + \frac{PC}{AB} = \frac{MQ}{LN} + \frac{MP}{LN}$

or, $\frac{PQ}{AB} = \frac{MQ+MP}{LN}$ ......(6)

But, $PQ = v_r - v_b$

Let AB = d is diameter of circle of least confusion

LN = D, the diameter of the lens.

$MQ + MP = v_r + v_b = 2v$ (approximately)

Substituting these values on (5) to get

$$\frac{v_r - v_b}{d} = \frac{2v}{D}$$

$$\therefore d = D\frac{v_r - v_b}{2v} = D . \frac{\omega}{f} \cdot \frac{v^2}{2v} = \frac{1}{2} . D\omega \frac{v}{f}.$$

If parallel beam of light is incident on the lens $v = f$

$$d = \frac{1}{2} D\omega . \frac{f}{f}$$

$$\Rightarrow \boxed{d = \frac{1}{2} D\omega}$$

Hence, the diameter of circle of least confusion is independent on focal length of lens

5. **A glass clad fibre is made with core glass of refractive index 1.5 and cladding is doped to give a fractional index difference of 0.005. Find (i) the cladding index (ii) The critical internal reflection angle (iii) The external critical acceptance angle (iv) Numerical aperture (v) Acceptance angle.**

⇒ Refractive index of core $(n_1) = 1.5$

Refractive index of cladding $(n_2) = ?$

(i) $\Delta = \frac{n_1 - n_2}{n_1}$

or, $0.005 \times 1.5 = 1.5 - n_2$

or, $n_2 = 1.4925$

(ii) Critical internal reflection angle is

$$\sin C = \frac{n_2}{n_1} = \left(\frac{1.4925}{1.5}\right)$$

$$\text{or, } C = \sin^{-1}\left(\frac{1.4925}{1.5}\right) = 84.3^\circ$$

(iii) External critical acceptance angle is given by

$$n_0 \sin C = n_1 \sin (90 - 84.3)$$

$$\sin C = \frac{n_1}{n_0} \sin 5.7$$

$$\text{or, } C = \sin^{-1} (1.5 \sin 84.7) \qquad \because n_0 = 1 \text{ for air}$$

$$\therefore C = 8.56^\circ$$

(iv) $NA = n_1\sqrt{2\Delta} = 1.5\sqrt{2\times 0.005} = 0.15$

(v) Acceptance angle, $i = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

$$= \sin^{-1}\left(\sqrt{1.5^2 - 1.4925^2}\right)$$

$$= 8.6^\circ.$$

6. **A parallel beam of light (λ = 5890 A°) is incident on a thin glass plate (μ =1.5) such that the angle of refraction is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.**

⇒ Wavelength of light (λ) = 5890 A° = 5890×10^{-10} m

Refractive index of plate (μ) = 1.5

Angle of refraction (r) = 60°

Thickness of plate (t) = ?

For dark fring due to reflective light

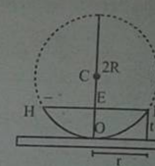
$$2\mu t \cos r = n\lambda$$

$$\therefore t = \frac{5890\times10^{-10}}{2\times1.5\times\cos 60^\circ} = 5.726\times10^{-7}\text{ m}$$

7. **How are Newton's rings formed? How is the ring diameter and film thickness related? How can Newton's rings experiment be used to determine refractive index of a liquid?**

⇒ Newton's rings are formed by enclosing thin air film of varying thickness between plane glass plate and convex lens of large radius of curvature due to the interference of light.

Let R be radius of curvature of plano convex lens, t is thickness of air film and r is radius of ring as shown in figure.



From geometry, HE. EP = OE (2R – OE)

$$\text{or, } r.r = t.\ 2R$$

where 2R – OE ≈ 2R approximately.

$$\text{or, } r^2 = 2t\,R$$

$$\text{or } t = \frac{r^2}{2R} = \frac{D^2}{8R}$$

$$\text{i.e., } t = \frac{D^2}{8R}$$

This is the relation between ring diameter and thickness of air film.

For next part refer to Q. N. 4 of 069 Chaitra

OR,

What is double refraction? How can we experimentally distinguish between plane polarized, circularly polarized and elliptically polarized light?

⇒ **Double refraction**

When an ordinary unpolarized light is incident on calcite or quartz crystal, the crystal splits the refracted rays into ordinary and extra-ordinary ray. This phenomenon is called double refraction.

Experimentally, plane polarized circularly polarized and elliptically polarized light can be distinguished as follows.

Plane polarized light

If the intensity of incident light changes and vanishes when passed through Nicol prism, then the incident light is said to be plane polarized.

Circularly polarized light

First pass the light through quartz wave plate and then analyze by rotating Nicol prism. If its intensity changes and vanishes, then it is said to be circularly polarized.

Elliptically polarized

Pass the incident light through Nicol prism. If its intensity changes but does not vanishe completely, then again passe this through quarter wave plate and analyze by Nicol prism. Then, such a light is called elliptically polarized light.

8. **Assume that the limits of the visible spectrum are arbitrary chosen as 430 nm and 680 nm. Calculate the no. of rulings per millimeter of a grating that will spread the first-order spectrum through an angle of 20°.**

⇒ Angle of diffraction $(\theta_1) = 20^\circ$

For $\lambda_1 = 430 \times 10^{-9}\ m = 430 \times 10^{-6}\ mm$

Using, $(a + b) \sin\theta_1 = n\lambda_1$

$$\text{or, } (a + b) = \frac{430 \times 10^{-6}}{\sin 20^\circ} = 1257.2 \times 10^{-6}\ mm$$

$$\therefore \text{Number of lines, } N = \frac{1}{a+b} = 795\ \text{lines/mm}$$

For $\lambda_2 = 680\ nm = 680 \times 10^{-6}\ mm$

$$(a + b) \sin\theta = n\lambda_2$$

$$\text{or, } a + b = \frac{680 \times 10^{-6}}{\sin 20^\circ} = 1988 \times 10^{-6}\ mm$$

$$\therefore N = \frac{1}{a+b} = 502\ \text{lines/mm}$$

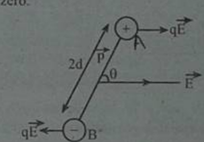
Hence, no. of lines varies from 502 to 795 per millimeter.

9. **Define an electric dipole. How does a dipole behave in electric field? Obtain the conditions for maximum torque and maximum potential energy in an electric field.**

⇒ **Electric dipole**

Two equal and opposite point charges separated by a small distance is called an electric dipole. The dipole moment of an electric dipole is defined as the product of one of the charges and the separation of charges i.e., p = 2qd. The quantity p behaves like a vector, directed from the negative to positive charge along the line joining the two charges.

Let on electric dipole AB of separation 2 d and magnitude of charge q placed at field $\vec{E}$. Dipole moment $\vec{p}$ makes an angle θ with $\vec{E}$. At equilibrium condition, at two ends of charges, electrostatic force F_A & F_B acts on opposite direction with same magnitude qE. Thus, net force acting on dipole is zero.



Total torque acting on dipole is

$$\tau = \text{Force} \times \text{perpendicular distance}$$
$$= q.E.\ d \sin\theta + qE \times d \sin\theta$$
$$= 2d\ q\ E \sin\theta$$
$$\text{or, } \tau = p\ E \sin\theta \quad [p = 2qd]$$

In vector form,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

For maximum torque experienced on dipole, $\vec{p}$ and $\vec{E}$ should be perpendicular.

When a dipole changes direction, electric torque does work on it with corresponding change in potential energy

The work done dw by torque τ during an infinitesimal displacement is

$$dw = \tau\, d\theta$$

The torque is in the direction of decreasing θ i.e.,

$$\tau = -pE \sin\theta$$

$$\therefore dw = -\tau d\theta = -pE \sin\theta\, d\theta$$

Total work done is

$$w = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$

$$w = pE \cos\theta_1 - pE \cos\theta_2$$

For maximum potential energy, dipole turned through θ_1 = 180° to θ_2 = 0°

i.e., w = pE cos 0° − pE cos 180°

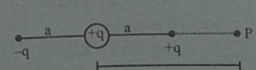
$$\boxed{w = 2pE}$$

which is the required amount of maximum potential energy.

OR,

For the charge configuration of the figure, snow that V(r) at a point P on the line assuming r >> a is given by $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right)$

⇒



Potential at point P due to charge −q is

$$V_1 = \frac{-q}{4\pi\varepsilon_0(r+a)} \quad \text{..............(i)}$$

Potential due to charge +q at distance r from P is

$$V_2 = \frac{q}{4\pi\varepsilon_0 r} \quad \text{..............(ii)}$$

Potential due to charge q at distance r − a from P is

$$V_3 = \frac{q}{4\pi\varepsilon_0(r-a)} \quad \text{..............(iii)}$$

Total potential at point P is

$$V = V_1 + V_2 + V_3$$

$$= \frac{-q}{4\pi\varepsilon_0(r+a)} + \frac{q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0(r-a)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-r(r-a) + (r-a)(r+a) + r(r+a)n}{r(r-a)(r+a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-r^2 + ra + r^2 - a^2 + r^2 + ra}{r(r^2-a^2)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - a^2 + 2ra}{r(r^2-a^2)}\right]$$

Since r >> a,we neglect a^2 that results

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right)$$

10. **A long cylindrical conductor has length 1 m and is surrounded by a co-axial cylindrical conducting shell with inner radius double that of long cylindrical conductor. Calculate the capacitance for this capacitor assuming that there is vacuum in space between cylinders.**

⇒ Let a = x and b = 2x be the inner and outer radii of cylindrical conducting shell.

Capacitance, $C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$

$$= \frac{2\pi\varepsilon_0 \times 1}{\ln 2}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\ln 2}$$

$$= 80.22 \text{ pF}$$

11. Charges of uniform volume density $3.2\mu C/m^3$ fill a non conducting solid sphere of radius 5 cm. What is the magnitude of the electric field at (a) 3.5 cm (b) 8 cm from the centre of the sphere?

⇒ Volume charge density (ρ) = $3.2\ \mu C/m^3$

Radius of sphere (R) = 5 cm = 0.05 m

Charge in sphere $(Q) = \frac{4}{3}\pi R^3.\rho$

$$= \frac{4}{3}\pi (5 \times 10^{-2})^3 \times 3.2 \times 10^{-6}$$

$$= 1.67 \times 10^{-9} \text{ C.}$$

a. Electric field, $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

At r = 3.5 cm,

$$E = \frac{1.67 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times (3.5 \times 10^{-2})^2} = 1.22 \times 10^4 \text{ N/C}$$

b. $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

At r = 8 cm,

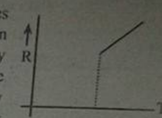
$$E = \frac{1.67 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times (8 \times 10^{-2})^2} \text{ N/C} = 2.34 \times 10^3 \text{ N?C}$$

12. What are superconductors? How they differ from perfect conductors? Give basic properties and uses of superconductors.

⇒ **Super conductors**

The electrical resistance of metals & alloys decreases as the temperature is lower. If we study the variation of resistance with temperature, it is found that at very low temperature,

the resistance becomes immeasurable. This phenomenon in which the electrical resistivity suddenly drops to zero when the material is cooled to a sufficiently low temperature is called super conductivity. The material is known as super conductor.



Difference between superconductor and conductor.

If superconductor is first cooled to a temp $T < T_C$ and then magnetic field is applied of the conductor is first cooled to a temperature T = 0 K and then magnetic field is applied, in such case no field enters the perfect conductor and super conductor.

However, if magnetic field is first applied at $T > T_C$ in case of super conductor and T > O (ρ = 0) in case of perfect conductor, their behaviour is different. In superconductor, flux does not penetrate, But if the magnetic field is applied to a conductor at T > 0 K (when $\rho \neq 0$) and then cooled to 0^- K (ρ = 0), the field remains as it is and even when the external magnetic field is removed, the field remains frozen in the material and can't change.

Properties of super conductor.

i. Effect of temperature: If a ring of super conducting material is cooled in a magnetic field from a temperature above transition temperature T_C to below T_C, and then magnetic field is switched off, an induced current is set up in the ring. This current continues undiminished for a very long time and is known as persistent current.

ii. Isotope effect: Critical temperature of super conductor varies with isotopic mass $T_C \sim M^{-1/2}$

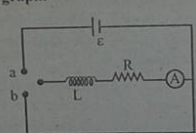
iii. Thermal conductivity of material changes discontinuously during the transfer from normal to super conducting state

iv. Impurities modify almost all the super conducting properties.

Uses of superconductor

1. It has application in generation & transmission of electric power.
2. It is used in supercomputers
3. It is used in magnetically levitating world's fastest train
4. Medical diagnosis

13. **Derive the relation for rise and fall of current in LR circuit. Plot a graph between current and time and explain the graph.**



⇒ **Rise of current in LR circuit**

Consider L and R be the inductance and resistance of inductor and resistor and ε be the emf of the source.

If inductor were not present, current would quickly rise to steady value $\frac{\varepsilon}{R}$.

When the switch is towards a current flow anticlockwise direction than we can apply kirchhoff's loop rule to this circuit.

$\varepsilon = V_L + V_R$ ....(i)

or, $\varepsilon = \frac{L\,di}{dt} + iR$

or, $\frac{L\,di}{dt} + iR - \varepsilon = 0$

or, $\frac{L\,di}{dt} + R(i - \varepsilon/R) = 0$

or, $\frac{L\,di}{dt} + R(i - i_0) = 0$ [Where $i_0 = \varepsilon/R$ is maximum current.]

or, $\frac{di}{i - i_0} = -\frac{dt}{\tau_L}$, where $\tau_L = \frac{L}{R}$ is inductive time constant

Integrating, we get

$$\int_0^i \frac{di}{i - i_0} = -\frac{1}{\tau_L}\int_0^t dt$$

$$\ln(i - i_0) + k_1 = \frac{-t}{\tau_L} + Kk_2$$

where k_1 and k_2 are integrating constants.

Using initial conditions that at $i = 0$, $t = 0$, we get

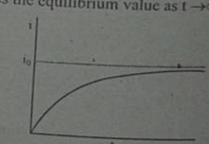
$k_1 = -\ln i_0$ and $k_2 = 0$

$$\therefore \ln\left(\frac{i - i_0}{i_0}\right) = \frac{-t}{\tau_L}$$

or, $\left(\frac{i - i_0}{i_0}\right) = e^{\frac{-t}{\tau_L}}$

or, $i = i_0 (1 - e^{\frac{-t}{\tau_L}})$

This is the expression for rise of current. If $t \to \infty$, $e^{\frac{-t}{\tau_L}}$ ... so current initially increased very rapidly and then gradually approaches the equilibrium value as $t \to \infty$.



Thus, an inductor acts to oppose changes in current through it. A long time later, it acts like ordinary conducting wire.

Fall of current in LR circuit

Now, if switch s is thrown from a to b as the current reaches i_0, then decaying of current starts due to absence of battery in the circuit. Applying Krichhoff's rule results uctive time constant.

nstants.

0, t = 0, we get

current. If $t \to \infty$, $e^{\frac{-t}{\tau_L}} \to 0$,
y rapidly and then gradually
as $t \to \infty$.

e changes in current through
ordinary conducting wire.

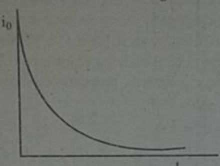
n a to b as the current reaches
ts due to absence of battery in
's rule results

$$iR + L\frac{di}{dt} = 0$$

Solution of this differential equation is

$$i = i_0 e^{\frac{-t}{\tau_L}}$$

The decay of current is shown in figure.



decay of current in LR circuit

OR,

In a Hall-effect experiment, a current of 3A sent length wise through a conductor 1 cm wide, 4cm long and 10 μm thick produces a transverse (across the width) Hall potential differences of 10 μV when a magnetic field of 1.5T is passed perpendicularly through the thickness of conductor. From these data, find: (a) The drift velocity of the charge carrier, and (b) The number density of charge carrier.

⇒ Current (I) = 3A

Hall voltage (V_H) = 10μV

Thickness (t) = 10 μm

Magnetic field (B) = 1.5 T

Magnetic drift velocity (v_d) = ?

No. density of charge carrier (n) =?

We have, $V_H = \frac{Bi}{net}$

$$\text{or, } n = \frac{Bi}{e.t.V_H} = \frac{1.5\times3}{1.6\times10^{-19}\times10\times10^{-6}\times10\times10^{-6}}$$

$$= 2.8 \times 10^{29}/m^3$$

We have, $I = v_d e n A$

$$\text{or, } v_d = \frac{I}{neA}$$

$$= \frac{3}{2.8\times10^{29}\times1.6\times10^{-19}\times(1\times10^{-2}\times10\times10^{-6})}$$

$$= 0.67\times10^{-3} \text{ m/s}$$

14. A particular cyclotron is designed with flees of radius R=75 cm mid with magnets that can provide a field of 1.5 T. (i) To what frequency should oscillator be set if deuterons are to be accelerated? (ii) What is the maximum energy of deuterons that can be obtained? Given mass of the deuteron is 3.34×10^{-27} kg.

⇒ Radius of Dee (R) = 75 cm = 0.75 m

Magnetic field (B) = 1.5 T

Mass of Deuteron (m) = 3.34×10^{-27} kg

i. $f = \frac{Be}{2\pi m} = \frac{1.5\times1.6\times10^{-19}}{2\pi\times3.34\times10^{-27}} = 11.43\times10^6$ Hz

ii. Maximum energy $E = 2\pi^2 R_m^2 f^2 m$

$$= 2\times\pi^2\times0.75^2\times(11.43\times10^6)^2\times3.34\times10^{-27}$$

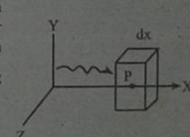
$$= \frac{0.48\times10^{-15}}{1.6\times10^{-13}}$$

$$= 30.3 \text{ MeV.}$$

15. Define Poynting vector. Prove that $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$

⇒ **Poynting vector**

The rate of flow of energy in an electromagnetic wave per unit area is described by a vector $\vec{S}$, known as Poynting vector.



When an electromagnetic wave propagates, there is transfer of electric & magnetic energy.

Consider an electromagnetic wave travelling in the right as shown in fig. The total energy at any instant dt stored in a box while travelling along positive X–axis is

$du = du_E + du_B = (u_E + u_B)\, dv = (u_E + u_B)\, A.dx$

For free square, $u_B = u_E$

$$du = 2u_E\, Adx = 2\frac{1}{2}\, \varepsilon_0 E^2 A.dx$$

$$\text{or, } \frac{du}{A} = E_0 E^2 dx$$

$$\text{or, } \frac{du}{A} = \varepsilon_0 E\, BC\, dt \qquad \because \frac{E}{B} = C$$

$$\text{or, } \frac{du}{Adt} = \frac{EB}{\mu_0} \qquad \because c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\text{or, } S = \frac{EB}{\mu_0}$$

In vector notation,

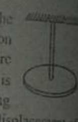
$$\boxed{\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}} \text{ proved.}$$

16. Prove that the energy levels are quantized, when the electron is confined in an infinite potential well of width *l*.

⇒ *Refer to Q.N. of 069 Poush*

2069 Poush

1. Derive the relation for the time period of a torsional pendulum and write the technique to find the moment of inertia of a body using torsional pendulum.



⇒ Consider a body such as disc suspended at the end of the wire, the other end is suspended on rigid support. When the disc is rotated, the wire is twisted by an angle θ., So, restoring torque is created on it obeying Hook's law. The restoring torque is directly proportional to the angular displacement of the wire.

i.e., $\tau \propto \theta$

or, $\tau = -C\,\theta$ ...... (1) where C is torsional constant

According to rotational dynamics,

$$\tau = I\,\alpha$$

$$\text{or, } \tau = I\frac{d^2\theta}{dt^2} \quad ......(2)$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

This is the equation of simple angular harmonic oscillator with an angular frequency $\omega = \sqrt{\frac{C}{I}}$

And time period $\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}}$ ....(3)

Let one regular body whose moment of inertia I_1 is known is placed on disc. Then, time period is measured as

$$T_1 = 2\pi\sqrt{\frac{I + I_1}{C}} \quad ....(4)$$

Similarly, if one irregular body of unknown moment of inertia I_2 is placed on disc, then time period is

$$T_2 = 2\pi\sqrt{\frac{I + I_2}{C}} \quad .....(5)$$

period of a torsional
to find the moment of
endulum.

ded at the
pended on
d, the wire
g torque is
e restoring
angular displacement of

orsional constant.

lar harmonic oscillator

....(3)

of inertia I_1 is known is
measured as

f unknown moment of
period is

From equations (3) and (4)

$$\frac{T_1^2}{T^2} = \frac{I + I_1}{I}$$

$$\frac{T_1^2 - T^2}{T^2} = \frac{I_1}{I} \quad \text{........(6)}$$

From equations (3) and (5)

$$\frac{T_2^2}{T^2} = \frac{I + I_2}{I}$$

$$\frac{T_2^2 - T^2}{T^2} = \frac{I_2}{I} \quad \text{.......(7)}$$

From (6) and (7), we write

$$\frac{T_2^2 - T^2}{T_1^2 - T^2} = \frac{I_2}{I_1}$$

$$\boxed{\therefore I_2 = \frac{T_2^2 - T^2}{T_1^2 - T^2} I_1} \quad \text{.............(8)}$$

By using equation (8), we can find the moment of inertia of unknown body..

For other portion refer to Q.N. 2070

2. **A string has linear density 525 gram/m³ and tension 45N. When sinsoidal wave of frequency 120 Hz and amplitude 8.5min is sent along the string, at what average rate does the wave transport energy?**

⇒ Here, linear density (μ) = 525 g/m

Tension (τ) = 45 N

Frequency of wave (f) = 120 Hz

Amplitude (y_m) = 8.5 m = 8.5×10^{-3} m

$$\omega = 2\pi f = 2\pi \times 120 = 754 \text{ rad/s.}$$

velocity of wave, $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{45}{0.525}} = 9.26$ m/s

Average power transferred is $P_{av} = \frac{1}{2} \mu v \omega^2 y_m^2$

$$\therefore P_{av} = \frac{1}{2} \times 0.525 \times 9.26\,0 \times 754 \times 0.0085 = 100 \text{ watt.}$$

3. **You are an engineer and would like to design a sound friendly auditorium hall in your city. What suggestions would you provide the concerned authority in order to incorporate**

a. Structural design

b. Materials to be used

c. Reverberation

⇒

a. Structural design

Distribution of intensity throughout the hall should be uniform. Usual design of the hall should be parabolic shape. At the speaker's end as shown each syllable of sound should be distributed in such a way that each word is heard distinctly and there is no reinforcement causing the change in quality. Coved wall or corners should be bare minimum because there may be unduly concentration of sound at some place and some other place may becomes zone of silence.

b. Materials to be used

In friendly auditorium, there should be provision to absorb the unnecessary reflected sound. Some porous material like cloths, asbestos, cushions, etc. should be used and they are put at various places of the hall. Open window is considered as perfect absorber of sound since there is no reflection and sound simply passes through. So, to get option value of reverberation time, we should have few windows.

c. Reverberation

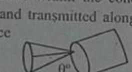
Interval of time taken by continuous sound to fall to an intensity by 60 decibles in loudness is called reverberation

Fall intensity of sound in hall is exponential.So, it will take longer time to become zero. Due to multiple reflection of wall, ceiling and floor sound reverberates and persist in side room for longer time. For good acoustic, reverberation time should have optimum value (not too small not too large).If small, sound vanishes instantaneously and gives the hall dead effort. If too large, there is multiple reflection and overlapping thereby causing confusion. Different frequency of sound may interfere differently at some point. So, quality of sound may change. This produces unpleasant effect, especially in time of music.

These are some suggestions to make sound friendly auditorium hall.

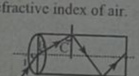
4. **What do you mean by acceptance angle and numerical aperture? Show that numerical aperture (NA) is proportional to square root of fractional refractive index change (Δ).**

⇒ **Acceptance angle:** It is defined as the maximum angle that a light ray can have relative to the axis of fiber and propagate down the fibre. The light rays contained within the cone having a full angle $2\theta_0$ are accepted and transmitted along the fibre. The cone is called acceptance cone.



Numerical aperture (NA): It is the light gathering ability of the fibre and it measures the amount of light accepted by the fibre.

Consider n_1 and n_2 be refractive indices of core and cladding of fibre, n_0 is refractive index of air.



Using Snell's law at interface of air and core

$$n_0 \sin i = n_1 \sin \theta \text{ ....(1)}$$

For core cladding interface

$$n_1 \sin C = n_2 \sin 90° \text{ ....(2)}$$

From (1), $\frac{n_0}{n_1} \sin i = \sin \theta = \text{sn}\,(90° - C) = \cos C$

$\therefore \text{cs}\, C = \frac{n_0}{n_1} \text{Sin}\, i$ ....(3)

Squaring and adding (2) & (3)

$$\sin^2 C + \cos^2 C = \frac{n_0^2}{n_1^2} \sin^2 i + \frac{n_2^2}{n_1^2} \sin^2 90°$$

$$\text{or, } 1 = \frac{n_0^2}{n_1^2} \sin^2 i + \frac{n_2^2}{n_1^2}$$

$$\text{or, } \frac{n_0^2}{n_1^2} \sin^2 i = \frac{n_1^2 - n_2^2}{n_1^2}$$

$$\text{or, } \sin^2 i = \frac{n_1^2 - n_2^2}{n_0^2}$$

$$\text{or, } \sin i = \sqrt{n_1^2 - n_2^2} \qquad \because \text{for air, } n_0 = 1.$$

Since NA = sin i, we write $NA = \sqrt{n_1^2 - n_2^2}$.

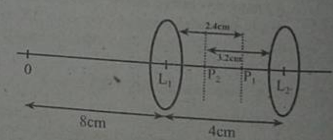
Fractional refractive index is expressed as

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{n_1^2 - n_2^2}{n_1 (n_1 + n_2)} \approx \frac{(NA)^2}{2n_1^2}$$

$$\boxed{\therefore NA = n_1 \sqrt{2\Delta}} \text{ proved.}$$

5. **Two thin converging lenses of focal lengths 6cm and 8cm are placed co-axially in air and are separated by 4cm. An object is placed 8cm in front of the first lens. Find the position, nature of final image.**

⇒



Here, $f_1 = 6$ cm, $f_2 = 8$ cm, $d = 4$ cm

$= \sin(90° - C) = \cos C$

& (3)

$\sin^2 i + \frac{n_2^2}{n_1^2} \sin^2 90°$

∵ for air, $n_0 = 1$.

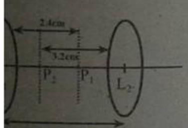
$NA = \sqrt{n_1^2 - n_2^2}$.

is expressed as

$\approx \frac{(NA)^2}{2n_1^2}$

roved.

ses of focal lengths 6cm and 8cm ir and are separated by 4cm. An front of the first lens. Find the mage.



$d = 4$ cm

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{6 \times 8}{6+8-4} = \frac{48}{10} = 4.8 \text{ cm}$$

$$\alpha = \frac{f.d}{f_2} = \frac{4.8 \times 4}{8} = 2.4 \text{ cm}$$

$$\beta = -\frac{f.d}{f_1} = -\frac{4.8 \times 4}{6} = -3.2 \text{ cm}$$

Object distance (U) $= -(u + \alpha) = -(8 + 2.4) = -10.4$ cm

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{V} + \frac{1}{10.4} = \frac{1}{4.8}$$

$$\frac{1}{V} = \frac{1}{4.8} - \frac{1}{10.4} \Rightarrow V = \frac{10.4 \times 4.8}{10.4 - 4.8} = 8.9 \text{ cm.}$$

Position of final image from second lens is $v = V + \beta$

$= 8.9 - 3.2 = 5.7$ cm

Since image distance is +ve, the image is real.

6. **Show that the intensity of second primary maxima of Franuhoffer's single slit diffraction is (1/62) of its central maxima.**

⇒ *Refer to Q.N. 3 of 070 Ashad*

OR,

What is double refraction? Prove that linearly and circularly polarized light are special cases of elliptically polarized light.

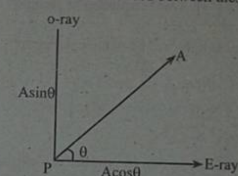
⇒ **Double refraction**

When an ordinary unpolarized light is incident on calcite crystal or quartz crystal, the crystal splits the refracted ray into ordinary ray and extra-ordinary ray. This phenomenon is called double refraction.

A point source of light in double refracting crystal is spherical because light travels with equal velocity in all direction. For extra-ordinary ray, the wavefront is ellipsoidal because velocity of light is different in different direction.

Along optic axis, the velocities of ordinary ray and extra ordinary ray are equal.

Suppose amplitude of incident plane polarized light on the crystal is A, it makes an angle θ with optic axis. Therefore amplitude of ordinary ray vibrating along PO is A sin θ and amplitude of extra-ordinary ray vibrating along PE is A cos θ. The phase of δ is introduced between these two rays.



For extra ordinary ray, $x = A \cos\theta \sin(\omega t + \delta)$ ....(1)

For ordinary ray, $y = A \sin\theta \sin\omega t$ ....(2)

Let $A \cos\theta = a$

$A \sin\theta = b$

Then $x = a \sin(\omega t + \delta)$ .....(3)

$y = b \sin\omega t$ ....(4)

From (4), $\frac{y}{b} = \sin\omega t$ and $\cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$

From equation (3), $\frac{x}{a} = \sin\omega t \cos\delta + \cos\omega t \sin\delta$

or, $\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$

On squaring, $\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$

or, $\frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\delta + \sin^2\delta) - \frac{2xy}{ab}\cos\delta = \sin^2\delta$

or, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\delta}{ab} = \sin^2\delta$ ......(5)

This is the general equation of ellipse.

Case I: W hen $\delta = 0$, equation (5) becomes

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\right) = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$\Rightarrow$ $\boxed{x = \frac{a}{b}y}$ This is equation of straight line. So, the emergent light will be plane polarized.

Case II: When $\delta = \frac{\pi}{2}$ and $a \neq b$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

This is the equation of symmetrical ellipse. The emergent light in this case will be elliptically polarized.

Case III: When $\delta = \frac{\pi}{2}$ and $a = b$

Then, equation (5) gives $x^2 + y^2 = a^2$

This represents equation of circle of radius 'a' and the emergent light will be circularly polarized.

Hence,linearly and circularly polarized light are special case of elliptically polarized light.

7. **A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the numbers of interference fringes per cm of the wedge length.**

$\Rightarrow$ Wave length of light $(\lambda) = 5.82\times10^{-7}$ cm

Wedge angle $(\theta) = 20 = \left(\frac{20}{60\times60}\right)^0 = \frac{20}{3600} \times \frac{\pi}{180}$ radian

We have, $\beta = \frac{\lambda}{2\theta} = \frac{5.82\times10^{-7}\times3600\times180}{2\times20\times\pi}$

$= 30011.53\times10^{-7}$

$= 0.00\,3001153$ m

$= 0.30011$ cm

Now, no. of fringes per cm is $\frac{1}{\beta} = \frac{1}{0.30011} = 3.$

8. **Find the slit separation of a double slit arrangement that will produce interference fringes 0.018 radian apart on a distance screen when the light has wavelength 589 nm?**

$\Rightarrow$ Wave length of light used $(\lambda) = 589$ nm $= 589\times10^{-9}$ m

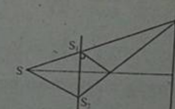
Angular fringe width $(\beta) = 0.018$ radian

Slit width $(d) = ?$

We have, $\beta = \frac{\lambda}{d}$

or, $d = \frac{\lambda}{\beta} = \frac{589\times10^{-9}}{0.018}$

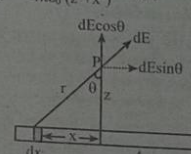
$= 3.3\times10^{-5}$m



9. **Charges are uniformly distributed on a long thin plastic scale. Calculate electric field intensity at an equilateral distance r from the centre of the scale.**

$\Rightarrow$ Let the scale be lying along X–axis and has uniform positive charge per unit length λ. The scale is divided into elementary segment of length dx. Electric field intensity due to this segment which is at distance x from far end of the rod is

$$dE = \frac{\lambda\,dx}{4\pi\varepsilon_0 r^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0 (z^2+x^2)} \quad \text{.....(1)}$$



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10. A spherica
(a) ρ = Ar
of the spher

⇒ Here charge

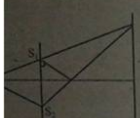
(i) Fig. shows
From Gauss

$\frac{1}{\beta} = \frac{1}{0.30011} = 3.$

double slit arrangement that inges 0.018 radian apart on a ht has wavelength 589 nm?

= 589 nm = 589×10^{-9} m

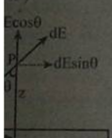
018 radian



tributed on a long thin plastic ield intensity at an equilateral of the scale.

X-axis and has uniform positive e scale is divided into elementary ectric field intensity due to this e x from far end of the rod is

.....(1)



This electric field makes an angle θ with vertical. So, it can be resolved into two components dE cosθ and dE sinθ. Taking similar charge element on the other side of the rod, we notice that electric field set up by this element also has magnitude dE, the components dE sin θ is cancelled out being equal and opposite. So, resultant electric field due to charged rod is

$$E = \int dE \cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int\frac{\cos\theta dx}{(x^2+z^2)}$$

For infinitely long rod,

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\cos\theta dx}{(x^2+z^2)}$$

$$\text{or, } E = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{z\,dx}{(x^2+z^2)^{\frac{3}{3}}} \qquad \left[\because \cos\theta = \frac{z}{\sqrt{z^2+x^2}}\right]$$

$$\text{or, } E = \frac{2\lambda}{4\pi\varepsilon_0}\int_{0}^{\infty}\frac{dx}{(x^2+z^2)^{\frac{3}{2}}}$$

$$\boxed{\therefore E = \frac{\lambda}{2\pi\varepsilon_0 z}}$$

This is the required expression for electric field intensity.

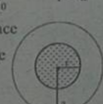
10. **A spherical charge distribution has volume charge density (a) $\rho = Ar^n$ at $r < a$ (b) $\rho = \rho_0$ for $r > a$, where a is the radius of the sphere and the electric field in both cases.**

⇒ Here charge density $\rho = A r^n$ $\quad r < a$

$= \rho_0$ $\quad r > a$

(i) Fig. shows Guassian surface

From Gauss' law, we have



$$\vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\int \rho\, dv$$

$$E.4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r Ar^n.\,4\pi r^2\, dr$$

$$E = \frac{1}{\varepsilon_0}r^2\int_0^r Ar^{n+2}\, dr = \frac{1}{\varepsilon_0}r^2\frac{Ar^{n+3}}{(n+3)}$$

$$\boxed{\therefore E = \frac{Ar^{n+1}}{E_0\,(n+3)}}$$

ii. $$\vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\int_0^a \rho\, dv$$

$$E = 4\pi\frac{1}{\varepsilon_0}r^2\int_0^a \rho_0\, 4\pi r^2\, dr$$

$$E = \frac{\rho_0\, 4\pi a^3}{4\pi\, 3\varepsilon_0 r^2}$$

$$\boxed{E = \frac{q}{4\pi\,\varepsilon_0 r^2}} \qquad \left[\because q = \rho_0 v = \frac{4}{3}\pi a^3 \rho_0\right]$$

11. **A neutral water molecule in its vapour state has an electric dipole moment of magnitude 7.1×10^{-30}cm. If the molecule is placed in an electric field of 2.5×10^4N/C, (i) What maximum torque can the field exert on it? (ii) How much work must an external agent do to turn this molecule end for end in this field?**

⇒ Dipole moment (P) = 7.1×10^{-30} cm

Electric field (E) = 2.5×10^4 N/C

(i) Maximum torque is given by

τ = P.E. sin 90°

$= 7.1\times10^{-30} \times 2.5\times10^4 = 17.75\times10^{-26}$ Nm

(ii) The work done by torque dw during infinitesimal displacement $d\theta$ is

$dw = \tau\, d\theta$

Total torque done is $w = \int_{\theta_1}^{\theta_2} dw$

$= \int_{0}^{180^\circ} PE \sin\theta \, d\theta$

$= -PE\,[\cos\theta]_0^{180}$

$= -PE\,[\cos 180^\circ - \cos 0^\circ]$

$= 2\,PE$

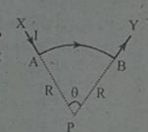
$= 2 \times 17.75 \times 10^{-26}$

$= 35.5 \times 10^{-26}$ Joule

12. **Calculate the displacement current between the capacitor plates of area 2.3×10^{-2} m^2 and rate of electric field change is 2.0×10^{12} V/ms. Also calculate the displacement current density and induced magnetic field for r = R = 70mm.**

⇒ *Refer to Q.N. 12 of 070 Bhadra*

13. **Derive the magnetic field at point P due to curve wire segment flowing current I as shown in below figure. ['R' is the radius of circle of arc AB]**



⇒ The given wire carrying current consist of three segments XA, AB, and BY.

Let B_1, B_2, and B_3 be magnetic fields at point P due to these three segments. The angle between $\vec{dl}$ and $\vec{r}$ is zero for straight sections XA and BY and thus, the product $\vec{dl} \times \vec{r} = 0$.

Hence, $B_1 = B_2 = 0$

However at every point on AB, $\vec{dl}$ is perpendicular to $\vec{r}$.
Using Biot-Savart's law,

$$dB_3 = \frac{\mu_0}{4\pi}\frac{Idl \sin 90^\circ}{R^2} = \frac{\mu_0 I\, dl}{4\pi R^2}$$

$$B_3 = \int dB_3 = \int_0^{\pi/2} \frac{\mu_0 I\, dl}{4\pi R^2} = \int_0^{\pi/2} \frac{\mu_0 I R d\theta}{4\pi R^2} = \frac{\mu_0 I}{4\pi R}\int_0^{\pi/2} d\theta$$

$$\therefore B_3 = \frac{\mu_0 I}{8R}$$

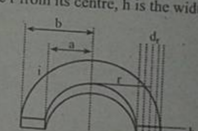
Thus, the total magnetic field at P due to the wire is

$$B = B_1 + B_2 + B_3 = \frac{\mu_0 I}{8R}$$

The direction of the field is into the plane of the wedge.

14. **Find an expression of the self inductance of a toroid having N number of turns, radius r and carrying current i.**

⇒ Let toroid of rectangular cross section area A carrying current i be considered. The external and internal radius of toroid are b and a. The elementary part of cross section be dr at distance r from its centre, h is the width of the strip.



Magnetic flux through the cross section is

$$\phi_B = \int \vec{B} \cdot \vec{dS} = \int_a^b B\, h\, dr$$

where $B = \frac{\mu_0 Ni}{2\pi r}\int_a^b \frac{dr}{r} = \frac{\mu_0 Ni}{2\pi r} \ln\left(\frac{b}{a}\right)$

is perpendicular to $\vec{r}$

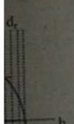
$\frac{R d\theta}{R^2} = \frac{\mu_0 I}{4\pi R} \int_0^{\pi/2} d\theta$

due to the wire is

he plane of the wedge.

lf inductance of a toroid radius r and carrying

s section area A carrying ernal and internal radius of y part of cross section be dr the width of the strip.



section is

$\ln\left(\frac{b}{a}\right)$

But inductance $L = \frac{N \phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$

This equation show that inductance depends only on geometric factors and number of turns.

15. **Write Maxwell's electromagnetic wave equation in dielectric medium. Obtain electromagnetic wave equations for E and B in both dielectric and free space.**

⇒ Maxwell's equation are

$$\left.\begin{aligned} &\nabla.\vec{D} = \rho \text{ ....i} \\ &\nabla.\vec{B} = 0 \text{ ....ii} \\ &\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ ......iii} \\ &\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \text{ ......iv} \end{aligned}\right\} \text{......(1)}$$

In dielectric medium $\rho = 0$, $\vec{j} = 0$, $\vec{D} = \epsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$

So, Maxwell's equation becomes

$$\left.\begin{aligned} &\nabla . \vec{E} = 0 \text{ ...i} \\ &\nabla . \vec{H} = 0 \text{ ...ii} \\ &\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \text{ ....iii} \\ &\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \text{ ......iv} \end{aligned}\right\} \text{......(2)}$$

Taking curl on both sides of equation 1(iii), we get

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$\text{or, } \nabla(\nabla . \vec{E}) - (\nabla . \nabla)\vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\text{or, } 0 - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left\{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right\}$$

$$\text{or, } \boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \text{ ....(3)}$$

This is the electromagnetic wave equation in free space.

Taking curl on both sides of equation 2(iii), we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$\nabla(\nabla . \vec{E}) - (\nabla.\nabla)\vec{E} = -\mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\text{or, } -\nabla^2 \vec{E} = -\mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\therefore \nabla^2 \vec{E} - \mu E \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \text{ ....(4)}$$

Again, for magnetic field in free space

From equation (1), $\nabla \times (\nabla \times \vec{H}) = \nabla \times \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\text{or, } \nabla(\nabla.\vec{H}) - (\nabla.\nabla)\vec{H} = \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\text{or, } 0 - \nabla^2 \vec{H} = \varepsilon_0 \frac{\partial}{\partial t}\left(\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$\text{or, } -\nabla^2 \vec{H} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or, } \boxed{\nabla^2 \vec{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0} \text{ .....(5)}$$

Again, taking equation 2(iv) for dielectric medium

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Taking curl $\nabla \times (\nabla \times \vec{H}) = \varepsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$

$$\nabla (\nabla . \vec{H}) - (\nabla . \nabla)\vec{H} = \varepsilon \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

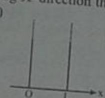
$$\text{or, } -\nabla^2 \vec{H} = -\varepsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\therefore \nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}} \quad ....(6)$$

Equation (3), (4), (5) & (6) are required expressions.

16. **A free particle is confined in a box of width L. Find an expression for energy eigen value and show that the particle can have only discrete energy.**

⇒ Consider a particle moving inside box along X–direction the box has infinite potential barriers at $x = 0$ and at $x = L$.



Potential function is

$$V = 0 \text{ for } 0 < x < L$$
$$= \infty \text{ for } x < 0 \text{ and } x > L \; ....(1)$$

The particle can't exist outside the box and so its wave function Ψ is 0 for $x < 0$ and $x > L$. Our task is to find what Ψ is within the box.

Within box, Schrödinger wave equation becomes

$$\frac{d^2\Psi}{dt^2} + \frac{2m(E-V)}{\hbar^2}\Psi = 0$$

$$\text{or, } \frac{d^2\Psi}{dt^2} + \frac{2mE}{\hbar^2} = 0 \; ....(2) \qquad \because V = 0 \text{ within box}$$

let $\frac{2mE}{\hbar^2} = k^2$ then

$$\frac{d^2\Psi}{dt^2} + k^2\Psi = 0$$

General solution of this equation is

$$\Psi = A \sin kx + B \cos kx \; ....(3)$$

The boundary conditions can be used to evaluate the constants A and B in equation (3)

$$\Psi = 0 \text{ at } x = 0 \rightarrow B = 0$$
$$\Psi = 0 \text{ at } x = L \rightarrow A \sin kL = 0$$

Since $A \neq 0$, $\sin kL = 0$

or, $kL = n\pi$

or, $k^2 L^2 = n^2 \pi^2$

or, $\frac{2mEL^2}{\hbar^2} = n^2\pi^2$

$$\boxed{\therefore E_n = \frac{n^2 \pi^2 \hbar^2}{2m L^2}}$$

For each n, there is an energy level, and corresponding wave function. Each value of E_n is called Eigen value and corresponding Ψ_n is called Eigen function.

Since $\hbar$, m, π, and L are fixed and value of n gives only discrete level of energy, we can conclude that the particle can have only discrete energy.

1. **Point out th**
oscillations
Show that th
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⇒ Similarities b
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ii. Both pend
inertia of a
iii. Both pend

Dissimilarities

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T = 2π √

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Differentiating

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s kx ....(3)

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= 0

sin kL = 0

nergy level, and corresponding wave of E_n is called Eigen value and led Eigen function.

are fixed and value of n gives only y, we can conclude that the particle energy.

1. **Point out the similarities and dissimilarities between the oscillations of bar pendulum and torsional pendulum. Show that the radius of gyration is equal to distance from center of suspension to center of gravity of compound pendulum when time period in minimum.**

⇒ Similarities between the oscillation of bar pendulum and torsional pendulum are

i. In both pendulum, the motions are simple harmonic.

ii. Both pendulum can be used to measure the moment of inertia of a body.

iii. Both pendulum oscillate about the centre of gravity.

Dissimilarities

i. Compound pendulum is used to measure acceleration due to gravity while torsional pendulum is used to measure modulus of rigidity.

ii. Amplitude of oscillation does not affect the motion of torsional pendulum while it affects the compound pendulum.

iii. There are two points having same time period in compound pendulum whereas there are not such two points in torsional pendulum.

The time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l_1} + l_1}{g}} \quad \text{.........(1)}$$

where k is distance from centre of suspension to centre of gravity of compound pendulum i.e., radius of gyration.

The time period will be minimum if $\frac{k^2}{l_1} + l_1$ is minimum.

Differentiating $L = \frac{k^2}{l_1} + l_1$ with respect to l_1, we get

or, $\frac{dL}{d\,l_1} = -\frac{k^2}{{l_1}^2} + 1$

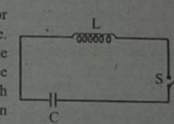
$\frac{dL}{d\,l_1} = 0 \Rightarrow k^2 = {l_1}^2$

or, $\boxed{k = \pm l_1}$ proved

$\therefore k = l_1$ taking +ve sign only for length

2. **Derive a differential equation for LC oscillation. Show that the maximum value of electric and magnetic energies stored in DC circuit are equal.**

⇒ Consider a circuit with capacitor and inductor as shown in figure. Let Q_0 be the maximum charge stored in capacitor. After the capacitor is fully charged, switch is closed. Then oscillation in circuit can be observed.



The total energy at any instant U in the oscillating circuit is the sum of electric and magnetic energies.

i.e. $U = U_{el} + U_{mag}$

$= \frac{Q^2}{2C} + \frac{1}{2}Li^2$

Differentiating with respect to time

$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{Q}{C}\frac{dQ}{dt}, \; i = \frac{dQ}{dt}, \frac{di}{dt} = \frac{d^2Q}{dt^2}$

Conservation of energy implies

$\frac{dU}{dt} = 0$

or, $L\frac{dQ}{dt}\cdot\frac{d^2Q}{dt^2} + \frac{Q}{C}\frac{dQ}{dt} = 0$

or, $L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$ This is the differential equation of LC oscillation.

Solution of this equation is

$Q = Q_0 \sin(\omega t + \phi)$ where $\omega = \frac{1}{\sqrt{LC}}$ is angular frequency.

Energy stored as electric field in capacitor at any time is

$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{2C}\sin^2(\omega t + \phi)$$

Energy stored in inductor as magnetic field is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}L\left(\frac{dQ}{dt}\right)^2 = \frac{1}{2}L\omega^2 Q_0^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2}L \cdot \frac{1}{LC} Q_0^2 \cos^2(\omega t + \phi)$$

$$= \frac{Q_0^2}{2C}\cos^2(\omega t + \phi)$$

When we take $\phi = 0$, at time $t = 0, T, 2T, \ldots$ etc.

$$U_E = 0 \text{ and } U_B = \frac{Q_0^2}{2C}$$

Thus, all energy is stored in electric field of capacitor.

And, at time $t = \frac{T}{2}, \frac{3T}{2}, \frac{5T}{2}, \ldots$

$$U_E = \frac{Q_0^2}{2C} \text{ and } U_B = 0$$

Thus, all energy is stored in magnetic field of inductor.

OR,

Prove that if a transverse wave is travelling along a string, then the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.

⇒ The general equation of S.H.M. is

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

Particle speed, $v_{particle} = \frac{dy}{dt} = \frac{2\pi va}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$

Also, slope of displacement curve is

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

$$\text{or, } \frac{dy}{dx} = \frac{v_{particle}}{v}$$

Hence, for a wave propagated on string the slope of displacement curve is the ratio of particle velocity to the wave velocity.

3. **The time of reverberation of an empty hall is 1.5 sec with 500 audiences present in the hall; the reverberation time falls to 1.4sec. Find the no. of persons present in the hall if the reverberation time falls down to 1.32 sec.**

⇒ *Refer to Q. N. 3. 070 Ashad*

4. **Show that the intensity of the first subsidiary maxima of Fraunhoffer's diffraction at a single slit is 4.5% of that of principal maxima.**

⇒ *Refer Q.N. 3. of 070 Ashad to derive*

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

where $\alpha = \frac{(2n+1)\pi}{2}$

For 1st subsidiary maximum, take $n = 1$

$$\therefore \alpha = \frac{3\pi}{2}$$

Intensity of 1st maximum is $I_1 = I_0 \frac{\sin^2\left(\frac{3\pi}{2}\right)}{\left(\frac{3\pi}{2}\right)^2}$

$$\therefore I_1 = 0.045\ I_0$$

Hence the intensity is 4.5% of principal maxima.

5. **In a Ne... the len... many b... (b) Ho... arrange...**

⇒

a. Rad... dian... radi... wav... We l... or, n

b. Whe... obtai... n =

6. **A diffrac... order at 3... total numb...**

⇒ Grating ele... For 2nd orde... (a + b) sin... (a + b) = 2...

Number of l...

7. **What is po... cannot occ... atomic level...**

⇒ **Population i...** The establis... atoms in the... energy level...

5. **In a Newton's ring experiment, the radius of curvature of the lens is 5cm and the lens diameter is 20mm. (a) How many bright rings are produced? Assume that $\lambda = 589nm$ (b) How many bright rings would be produced if the arrangement were immersed in water (μ=1.33)?**

⇒

a. Radius of curvature of lens (R) = 5cm.

diameter of lens (d) = 20 mm

radius of lens (r) = 10 mm = 1 cm

wave length of light used (λ) = 589nm = 589×10^{-7} cm

We know that, $D_n^2 = 4n\lambda R$

$$\text{or, } n = \frac{D_n^2}{4\lambda R} = \frac{2}{4\times589\times10^{-7}\times5} = 3396$$

b. When the lens is immersed in water, no. of fringes obtained will be

$$n = \frac{D_n^2}{4\mu\lambda R} = \frac{3396}{1.33} = 2553$$

6. **A diffraction grating 3cm wide produces the second order at 33° with light of wavelength 600nm. What is the total number of lines on the grating.**

⇒ Grating element (a + b) =?

For 2[nd] order diffraction, we write

$$(a + b) \sin\theta_2 = 2\lambda$$

$$(a + b) = \frac{2\times600\times10^{-7}}{\sin 33} = 2203.3\times10^{-7} \text{ cm}$$

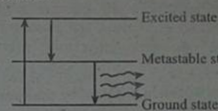
$$\text{Number of lines (N)} = \frac{1}{a+b} = 4538 \text{ lines/cm}$$

7. **What is population inversion? Explain why laser action cannot occur without population inversion between atomic levels?**

⇒ **Population inversion**

The establishment of a situation in which the number of atoms in the higher energy level is greater than that in lower energy level is called population inversion.

Laser works on the principle of stimulated emission which is possible only when the electrons jump from higher level to lower level by some trigger. In normal state number of atoms in lower energy level is higher than that in higher level. So, continuous transition from high level to low level is not possible.



Population inversion creates the number of atoms in higher state more so it is possible to jump these excited atoms to ground state by emitting radiation. Hence, the laser action cannot occur without population inversion between atomic levels.

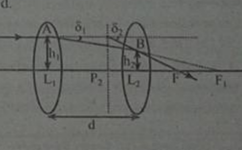
8. **What -are cardinal points of an optical system? Determine the equivalent focal length of a combination of two thin lenses separated by a finite distance.**

⇒ **Cardinal points**

There are six different points on principal axis which are considered as reference point to measure various distances in the refraction through a thick lens and in a system of coaxial lenses. These six points of reference are called cardinal points. They are:

i. Two principal focal points
ii. Two principal points
iii. Two nodal points

Let f_1 and f_2 be focal lengths of two lenses separated by distance d.



A ray parallel to axis refract along AB by lens L_1 and finally focus at F by lens L_2. AB meets at F_1 when produced forward.

Let δ_1 and δ_2 be the deviation produced by lens L_1 and L_2 respectively.

Then, $\delta_1 = \frac{h_1}{f_1}$ ....(i)

$\delta_2 = \frac{h_2}{f_2}$ ....(ii)

The deviation produced by equivalent lens is $\delta = \frac{h_1}{f}$ where f is the focal length of equivalent lens

$\delta = \delta_1 + \delta_2$ is the total deviations

or, $\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$ ....(iii)

From figure in similar Δs AL_1F_1 and BL_2F_1

$$\frac{AL_1}{BL_2} = \frac{L_1F_1}{L_2F_1}$$

$$\text{or, } \frac{h_1}{h_2} = \frac{f_1}{f_1 - d} \Rightarrow h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Substituting h_2 in equation (iii), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

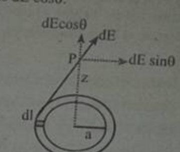
$$\boxed{\therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

This is the required expression for focal length of equivalent lens.

9. **A ring has a charge uniformly distributed in it. Derive an expression for electric field at any point on the axial line of the ring. Extend your result to find the potential.**

⇒ Consider a ring of radius a, charge q and linear charge density λ. Electric field at point P and at distance z from centre of the ring can be calculated by integrating the electric

field due to small segment of ring of length dl. Horizontal components $dE \sin\theta$ will be cancelled out being equal and opposite round the ring, so the field is only due to vertical components $dE \cos\theta$.



$$E = \int dE\cos\theta$$

where $dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0\sqrt{a^2+z^2}}$ and $\cos\theta = \frac{z}{\sqrt{a^2+z^2}}$

$$\text{or, } E = \frac{z\lambda}{4\pi\varepsilon_0(z^2+a^2)^{3/2}} \int_0^{2\pi a} dl$$

$$\therefore E = \frac{qz}{4\pi\varepsilon_0(z^2+a^2)^{3/2}} \text{ ....(1) } [\because q = 2\pi a\lambda]$$

We know that, $\vec{E} = -\nabla V$

or, $V = \int \vec{E} \cdot d\vec{z}$ for one dimension

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0}\int \frac{z\,dz}{(z^2+a^2)^{3/2}}$$

$$\boxed{\therefore V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+a^2}}}$$

which is the required expression for potential.

OR,

Write an expression for electric field at any point in the axial line of a charged ring. Using this equation, calculate the electric field at any point on the axial line of a charged disk.

g of length dl. Horizontal
celled out being equal and
eld is only due to vertical

sinθ

and $\cos\theta = \frac{z}{\sqrt{a^2+z^2}}$

∴ $q = 2\pi a\lambda$]

nension

potential.

ield at any point in the
this equation, calculate
n the axial line of a

⇒ *Refer to Q. N. of 070 Bhadra*

10. What is the magnitude of the electric field at the point (2, 3) m if the electric potential is given by $V = 2x + 5xy + 3y^2$ volts? What acceleration does an electron experience in the x-direction?

⇒ $V(x, y) = 2x + 5xy + 3y^2$

We have, $E_x = \frac{-\partial V}{\partial x} = -(2 + 5y)$

Electric field at (2, 3) is given as

$E_x(2, 3) = -(2 + 5 \times 3) = -17 V/m$

Force experienced by electron is $(F) = -e E_x$

or, $ma = 17 \times 1.6 \times 10^{-19}$

$\therefore a = \frac{17 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 2.98 \times 10^{12}\ m/s^2$

11. Derive an equation $\vec{J} = \sigma\vec{E}$. Explain why resistivity of a conductor increases with increasing temperature. Plot a graph between R (resistance at any temperature θ) and temperature. Based on the graph, explain, what are superconductors? How they differ from perfect conductors? Describe the characteristics of superconductors.

⇒ Consider an electron of mass m and charge e be in electric field $\vec{E}$. It experiences a force $\vec{F} = e\vec{E}$.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{e\vec{E}}{m}$$

If τ be average time between collision, the drift speed is

$v_d = a\tau$, then

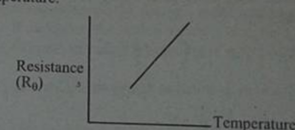
$$\vec{v_d} = \frac{e\vec{E}\tau}{m} \text{ (in vector form)}$$

Also, current density $(\vec{J}) = ne\vec{v_d}$

$$\therefore \vec{J} = \left(\frac{ne^2\tau}{m}\right)\vec{E}$$

$\vec{J} = \sigma\vec{E}$, where $\sigma = \frac{ne^2\vec{E}}{m}$ is conductivity of conductor.

In conductor, electric current is carried out by electrons. As the temperature increase, the vibrational motion of an electron will increase and hence, they disturb the flow of current through them i.e., resistivity increase with temperature.

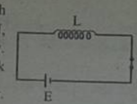


At low temperature, arrangement of electrons in conductor is in order, they do not lose energy during collision and vibration. So, current flow easily. For example at -4°k copper wire becomes super Conductor; it does not restrict the flow of electron.

Perfect conductor	Super Conductor
(1) It is not thermodynamically reversible.	(1) It is thermodynamically reversible.
(2) Meissner effect depends on history of applied magnetic field.	(2) Meissner effect does not depend on history of applied magnetic field.
(3) Its resistance is zero at all temperature.	(3) Its resistance is almost zero at low temperature.
(4) It is practically impossible.	(4) It is practically possible.

12. Derive an expression for energy stored in magnetic field. Show that the energy stored per unit volume is directly proportional to the situate of the magnetic field density. Compare this result with electric energy density.

⇒ Suppose an inductor is connected with a battery of emf ε as the switch is ON, current in the circuit rise continuously. While current is rising an induced back emf $\varepsilon = -L\frac{di}{dt}$ appears across inductor.



So the source must do some work against the back emf.

Rate of doing work is $P = i\varepsilon = Li\frac{di}{dt} \rightarrow \varepsilon = L\frac{di}{dt}$

Total work done in bringing current from zero to steady value I is

$$W_B = \int_0^t P\,dt = L\int_0^I i\,di = \frac{1}{2}LI^2$$

This work is stored as energy in inductor, so that

$$U_B = \frac{1}{2}LI^2$$

Energy stored per unit volume is given by

$u_B = \frac{U_B}{Al}$, where Al is volume of solenoid

$$= \frac{lLI^2}{2Al}$$

$$= \frac{lI^2}{2A}\cdot\mu_0 n^2 A$$

[since $\frac{L}{l} = \mu_0 n^2 A$ is inductance per unit length]

As $B = \mu_0 nI$, $u_B = \frac{1}{2\mu_0}(\mu_0 nI)^2$

$$\boxed{\therefore u_B = \frac{B^2}{2\mu_0}}$$ proved.

Thus, electric energy density per unit volume is given by $u_E = \frac{E^2}{2\varepsilon_0}$, The energy density in this case is also directly proportional to the square of the electric field.

OR,

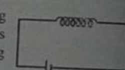
12. What is self-inductance? Define inductance of a coil. Show by calculation inductance of a coil depends on the permeability of a medium and the geometry of the coil.

⇒ **Self inductance**

It is defined as the property of a circuit due to which any change in the current flowing in the circuit induces an emf that opposes the change and then, the energy is stored in the magnetic field established by the current.

Inductance of a coil is the property of coil due to which any change in magnitude of the flux linked with it, induces an emf in it. The induced emf is called back emf.

Consider solenoid of length l having N number of turns, area of cross section be A, I be current flowing through it.



Flux through each turn $(\phi) = BA$

Total flux $(\phi_{total}) = \phi N = BAN = \frac{\mu_0 NI}{l}AN = \frac{\mu_0 N^2 IA}{l}$ ....(i)

From the definition of self inductance,

$\phi_{total} = LI$ .... (ii)

Equating (i) and (ii), we get

$$LI = \frac{\mu_0 N^2 IA}{l}$$

$$\boxed{\therefore L = \frac{\mu_0 N^2 A}{l}}$$

From this equation, it is clear that inductance depends an permeability μ of the medium and geometry i.e., area of coil and length of the coil.

13. A long circuit coil consisting of 50 turns with diameter 1.2m carries a current of 10Amp. (a) Find the magnetic field at a point along the axis 90cm from the center. (b)

At what distance fro... is 1/8 as great as at t...

⇒ Number of turns (N) ...

Radius of coil (R) = 0...

Current (I) = 10A

(a) We have, B = ...

(b) Let x be the dista...

$B_x = \frac{1}{8}B_{centre}$

or, $B_x = \frac{1}{8}B_0$

or, $\frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$ = ...

or, $x = \pm R\sqrt{3}$

$x = 0.6 \times \sqrt{3} = 1$...

∴ At distance 1.04 m, ... at the centre.

14. Describe the princip... that the time taken ... semicircle is exactly ... velocity.

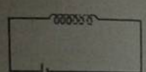
⇒ Cyclotron consists of ... semicircular metal bo... called dees. A source ... located near the midp... gap between the dees. ... are connected to a pow... frequency oscillator. ... apparatus is placed be... pole pieces of strong ele...

e? Define inductance of a coil. [...]uctance of a coil depends on the [...]m and the geometry of the coil.

[...]erty of a circuit due to which any [...]wing in the circuit induces an emf [...]and then, the energy is stored in the [...]d by the current.

[...]e property of coil due to which any [...] the flux linked with it, induces an [...]nf is called back emf.

[...]ngth l having [...]rea of cross [...]rrent flowing



[...]$\phi) = BA$

$BAN = \frac{\mu_0 NI}{l} AN = \frac{\mu_0 N^2 IA}{l}$ ....(i)

[...]elf inductance,

.... (ii)

[...] get

[...] is clear that inductance depends an [...]edium and geometry i.e., area of coil

[...]sisting of 50 turns with diameter [...] of 10Amp. (a) Find the magnetic [...]he axis 90cm from the center. (b)

At what distance from the center, along the axis, the field is 1/8 as great as at the center.

⇒ Number of turns (N) = 50

Radius of coil (R) = 0.6 m

Current (I) = 10A

(a) We have, $B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}} = \frac{4\pi\times10^{-7}\times50\times10\times0.6^2}{2(0.9^2+0.6^2)^{3/2}}$

$\therefore B = 284.4\times10^{-7}$ T

(b) Let x be the distance from the centre

$B_x = \frac{1}{8} B_{centre}$

or, $B_x = \frac{1}{8} B_0$

or, $\frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}} = \frac{1}{8}\frac{\mu_0 NIR^2}{2(0+R^2)^{3/2}}$

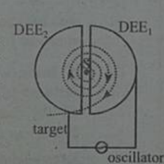
or, $x = \pm R\sqrt{3}$

$x = 0.6 \times \sqrt{3} = 1.04$ m

$\therefore$ At distance 1.04 m, the field magnitude is $\left(\frac{1}{8}\right)^{th}$ as great as at the centre.

14. Describe the principal and working of Cyclotron. Show that the time taken by the ion in a Dee to travel a semicircle is exactly same whatever be its radius and velocity.

⇒ Cyclotron consists of two hollow semicircular metal boxes D_1, D_2 called dees. A soarce of ions is located near the midpoint of the gap between the dees. The dees are connected to a powerful radio frequency oscillator. The whole apparatus is placed between the pole pieces of strong electromagnet.



Suppose positive ion leave the ion source at the centre of the chamber at the instant when dees D_1 and D_2 are at maximum –ve & +ve A.C potential respectively.

The +ve ion will be accelerated toward negative dees D_1 before entering it. The ion enter the space between dees with velocity given by

$Ve = \frac{1}{2}mv^2$, V is applied voltage

When ion is inside the dee, ion is not acted by electric field but it is under the action of magnetic field, it travels in circular path of radius is given by

$$Bev = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Be}$$

Angular velocity $(\omega) = \frac{v}{r} = \frac{Be}{m}$

Time taken by electron to travel semicircular path is

$$t = \frac{\pi}{\omega} = \frac{\pi m}{Be}.$$

From this equation, it is clear that time taken by electron to describe semicircle is independent to both radius of path and velocity of electron. Hence, the ion describes all semicircles whatever be their radii, in exactly the same time.

15. Write Maxwell's equations in free space and dielectric medium. With the help of Maxwell's equations, derive charge conservation theorem.

⇒ Maxwell's equations are

$\nabla . \vec{D} = \rho$ .... (i)

$\nabla . \vec{B} = 0$ .... (ii)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ .... (iii)

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ .... (iv)

In free space $\rho = 0$, $\vec{J} = 0$, $\vec{D} = \varepsilon_0 \vec{E}$, $\vec{B} = \mu_0 \vec{H}$

(a) Maxwell's equation in free space

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

(b) In dielectric medium

$$\nabla \cdot \vec{E} = 0$$

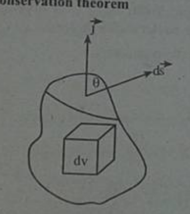
$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

According to the principle of conservation of charge, net amount of charge in an isolated system must remain constant. If the charge varies with time, the net charge leaving the volume crossing the boundary surface equals the net rate of flow of charge entering the volume.

Charge conservation theorem



Consider a surface elements $d\vec{s}$ of surface s enclosing a volume V, $\vec{J}$ be the current density directed as shown.

$\vec{J} \cdot d\vec{s}$ represents charge per unit time leaving the volume V across ds. Therefore, $\oint \vec{J} \cdot \vec{ds}$ represents charge per unit time leaving the volume V crossing the entire surface ?. Let q be instantaneous charge within volume.

Then, $\frac{-dq}{dt}$ measures the rate of decrease of charge within the volume.

Principal of conservation of charge gives

$$\oint \vec{J} \cdot \vec{ds} = \frac{-dq}{dt} = -\frac{d}{dt} \int \rho dv$$

Gauss divergence theorem is $\oint \vec{A} \cdot d\vec{s} = \int_{vol} \nabla \cdot \vec{A} \, dv$

or, $\int (\nabla \cdot \vec{J}) dv = -\int \frac{\partial \rho}{\partial t} dv$

or, $\int ((\nabla \cdot \vec{J}) + \frac{\partial \rho}{\partial t}) dv = 0$

Since the volume is arbitrary, integrand must vanish

Therefore, $\boxed{\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$

which is the principal of conservation of charge or continuity equation.

16. **A beam of electrons having energy of each 3eV is incident on a potential barrier of height 4eV. If the width of the barrier is 20A°, calculate the transmission coefficient of the beam through the barrier.**

⇒ Energy of electron (E) = 3eV

Barrier height (V) = 4eV

ements $d\vec{s}$ of surface s enclosing a
urrent density directed as shown.

arge per unit time leaving the volume
, $\oint \vec{J}.d\vec{s}$ represents charge per unit
me V crossing the entire surface $\vec{s}$.
charge within volume.

e rate of decrease of charge within the

on of charge gives

$$= \frac{-dq}{dt} = -\frac{d}{dt}\int \rho dv$$

em is $\oint \vec{A}.d\vec{s} = \int_{vol} \nabla.\vec{A}\, dv$

$$\int \frac{\partial \rho}{\partial t} dv$$

$) dv = 0$

bitrary, integrand must vanish

$= 0$

f conservation of charge or continuity

having energy of each 3eV is
barrier of height 4eV. If the width
A°, calculate the transmission
hrough the barrier.

eV

Barrier width (L) = 20 A° = 20×10^{-10} m

Transmission coefficient (T) =?

We have,

$$T = 16\frac{E}{V}\left(1 - \frac{E}{V}\right)\exp\left[-\frac{2L}{h}\sqrt{2m(V-E)}\right]$$

$$= 16\left(\frac{3}{4}\right)\left(1 - \frac{3}{4}\right)$$

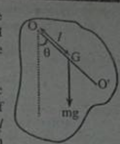
$$\exp\left[-\frac{2\times20\times10^{-10}}{1.05\times10^{-34}}\times\sqrt{2(9.1\times10^{-31}).(4-3)\times1.6\times10^{-19}}\right]$$

$$= 3.7\times10^{-9}$$

Since the transmission coefficient is very low, probability of transmission is very low.

2070 Ashad

1. **Derive an expression for the time period of a physical pendulum and establish the interchangeability of the center of oscillation and suspension.**



⇒ Consider a rigid object of mass m be suspended at point O. The centre of gravity of the object is G at distance l from O. Since l is the distance from the pivot to centre of mass, the restoring torque is $mgl \sin\theta$. From rotational dynamics, torque produced is

$$\tau = I\alpha \text{ ......(1)}$$

where I is moment of inertia of object about the given axis, α is angular acceleration and α is angular acceleration.

For balanced condition,

$$-mgl\sin\theta = I\frac{d^2\theta}{dt^2} \text{ ....(2)}$$

or, $\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0$ ...(3) [since for small θ, $\sin\theta \sim \theta$]

This is the equation for simple harmonic motion indicating that physical pendulum executes simple harmonic motion.

Moment of inertia of pendulum about its axis passing through O and perpendicular to it is given by

$I = mk^2 + ml^2$ .....(4) where k is the radius of gyration.

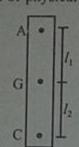
or, $\frac{d^2\theta}{dt^2} + \frac{mgl}{mk^2+ml^2}\theta = 0$ ....

Comparing this equation with differential equation of SHM, time period will be

$$T = 2\pi\sqrt{\frac{k^2+l^2}{lg}}$$

or, $T = 2\pi\sqrt{\dfrac{\frac{k^2}{l} + l}{g}}$

which is the required expression for time period of physical pendulum. The quantity $\frac{k^2}{l} + l$ is called equivalent length of the simple pendulum.



Interchangeability of point of suspension and oscillation: Let time period be T_1 for point A at distance l_1 from centre of gravity. Similarly, time period be T_2 for a point C at distance l_2 from centre of gravity as shown in fig.

$$T_1 = 2\pi\sqrt{\frac{\frac{k^2}{l_1} + l_1}{g}}, \quad T_2 = 2\pi\sqrt{\frac{\frac{k^2}{l_2} + l_2}{g}}$$

Also, $k^2 = l_1 l_2$

or, $\frac{k^2}{l_1} = l_2$ and $\frac{k^2}{l_2} = l_1$

Therefore, we have from above equation, $T_1 = T_2$.

This shows that point of suspension and oscillation are interchangeable.

2. **Give the necessary theory of forced vibration and deduce the condition for resonance amplitude.**

⇒ In actual practice, oscillation left to itself eventually die out after certain times. But by applying periodicexternal force, we can maintain a constant amplitude. The phenomenon of setting body into continuous vibrations with the help of strong periodic force having frequency different from the natural frequency of the body is called forced vibration.

Let the applied force be represented as

$$F_{ext} = F_0 \sin \omega t \text{ ....(1)}$$

Newton's 2nd law of motion in this case is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega t \text{ .....(2)}$$

Solution of such differential equation is

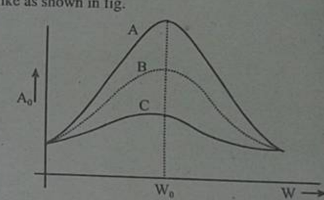
$$x = A_0 \sin(\omega t + \phi_0) \text{ .....(3)}$$

where ϕ_0 is phase angle between the displacement x and th external force F_{ext}. Here, the amplitude remains constan From (2) & (3), we have

$$A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2) + \frac{b^2\omega^2}{m^2}}} \text{ ....(4)}$$

$$\phi_0 = \tan^{-1}\left[\frac{w_0^2 - w^2}{w\left(\frac{b}{m}\right)}\right] \text{ .....(5)}$$

Amplitude of forced harmonic motion A_0 depends o difference between applied and natural frequency. If we plo A_0 as function of natural frequency ω, the graph will loo like as shown in fig.



Curve A is the case of light damping, B is the case of heav damping ,and C is the case of over damping. The amplitud becomes large when the applied frequency ω is near th natural frequency ω_0.

Hence, the resonance is defined as the phenomenon o setting body into vibration with its natural frequency by th application of external periodic force. Such frequency i called resonant frequency.

OR,

Show that the

oscillation is $\frac{1}{8}$

⇒ Let f and f' be t

Then, $f = \frac{1}{2\pi}\sqrt{}$

$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - }$

Fractional chang

Neglecting higher terms

$= \frac{1}{8Q^2}$ prov

+ kx = F_0 sin ωt .....(2)

ifferential equation is

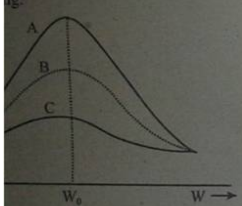
t + ϕ_0) .....(3)

angle between the displacement x and the ... Here, the amplitude remains constant. e have

$$\frac{F_0}{\sqrt{(\omega^2-\omega_0^2)+\frac{b^2\omega^2}{m^2}}}\ \ldots\ldots\ldots(4)$$

$$\left[\frac{\omega_0^2-w^2}{w\left(\frac{b}{m}\right)}\right]\ \ldots\ldots(5)$$

orced harmonic motion A_0 depends on en applied and natural frequency. If we plot f natural frequency ω, the graph will look ig.



ase of light damping, B is the case of heavy s the case of over damping. The amplitude hen the applied frequency ω is near the ω_0.

nance is defined as the phenomenon of vibration with its natural frequency by the ernal periodic force. Such frequency is quency.

OR,

Show that the fractional change in frequency of damped oscillation is $\frac{1}{8}Q^2$, where Q is quality factor.

⇒ Let f and f ' be the initial and final frequency of oscillation.

Then, $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ....(i) for free oscillation

$f'=\frac{1}{2\pi}\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$ ....(ii) for damped oscillation

Fractional change in frequency = $\frac{f-f'}{f}$

$$=\frac{\frac{1}{2\pi}\left(\sqrt{\frac{k}{m}}-\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}\right)}{\frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

$$=\sqrt{\frac{k}{m}}\left\{\frac{1-\sqrt{1-\frac{b^2m}{4m^2k}}}{\sqrt{\frac{k}{m}}}\right\}$$

$$=1-\sqrt{1-\frac{b^2}{4mk}}$$

$$=1-\sqrt{1-\frac{b^2}{4m(m\omega^2)}}\ \text{(since } k=m\omega^2\text{)}$$

$$=1-\sqrt{1-\frac{b^2}{m^2\omega^2}}$$

$$=1-\sqrt{1-\frac{1}{4Q^2}}\ \text{since } Q=\frac{m\omega}{b}$$

$$=1-\left(1-\frac{1}{2}\times\frac{1}{4Q^2}+\ldots\ldots\right)\ \text{using bionomial expansion}$$

Neglecting higher terms

$$=\frac{1}{8Q^2}\ \text{proved}$$

3. **The reverberation time for an empty hall is 1.5 sec. With 500 audiences present in the hall, the reverberation time falls to 1.4 sec. Find the number of persons present in the hall if the reverberation time falls down to 1.312 sec.**

⇒ Reverberation time is given by $T=\frac{0.165\ V}{\Sigma\alpha\ ds}$ .... (i) for empty hall

where V is volume of the hall and Σα ds is the sound absorbed by the surface.

or, $1.5=\frac{0.165\ V}{\Sigma\alpha\ ds}$

When 500 audience is present,

$T=\frac{0.165\ V}{\Sigma\alpha\ ds+500\alpha}$ ....(ii)

where α is aberration coefficient for single person and its value is nearly equal to 0.43.

or, $1.4=\frac{0.165\ V}{\Sigma\alpha\ ds+500\times0.43}$ ....(3)

Solving equation (ii) and (iii), we get Σα ds = 3010m^3 sabine and V = 27363.636 m^2

For reverberation time reduced to 1.312 sec, let's suppose that n be the number of audiences present in hall.

$T=\frac{0.165V}{\Sigma\alpha\ ds+n\ \alpha}$

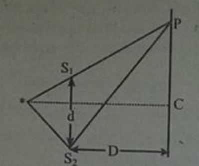
or, $1.312=\frac{0.165V}{\Sigma\alpha\ ds+n\times0.43}$

Putting value of V and Σα ds , we get

n = 1003

4. **What is interference? Explain the intensity distribution in interference with mathematical treatment.**

⇒ The modification of distribution of energy due to superposition of two light waves is called interference of light. These two light waves that interfere each other are called coherent sources.



Consider a monochromatic source of light S emitting waves of wavelength λ. S_1 and S_2 are two narrow pinholes equidistance from S and acts as two virtual coherent source. Let a be the amplitude of wave and δ be the phase difference between the two waves reaching the point P.

If y_1 and y_2 are displacements, then

$$y_1 = a \sin \omega t, \; y_2 = a \sin (\omega t + \delta)$$

$$\therefore \quad y = y_1 + y_2 = a \sin \omega t + a \sin (wt + \delta)$$

$$= a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$

$$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta$$

Let $a (1 + \cos \delta) = R \cos \theta$ ....(1)

$a \sin \delta = R \sin \theta$ ....(2)

$$\therefore y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

$$= R \sin (\omega t + \theta) \text{ ....(3)}$$

which represents equation of simple harmonic vibration of amplitude R.

Squaring and adding (1) and (2), we get

$$R^2 \sin^2\theta + R^2 \cos^2\theta = a^2 \sin^2 \delta + a^2 \cos^2\delta + a^2 + 2a^2 \cos\delta$$

or, $R^2 = 2a^2 + 2a \cos \delta$

or, $R^2 = 2a (1 + \cos\delta)$

or, $R^2 = 2a^2 \times 2 \text{Cos}^2\delta/2$

$\therefore R^2 = 4a^2 \cos^2\delta/2$

Intensity is the square of amplitude of resultant wave

$$I = R^2$$

$$I = 4a^2 \cos^2 \delta/2$$

Case I

For phase difference $\delta = 0, 2\pi, 4\pi, \ldots\ldots n(2\pi)$ or path difference $x = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots, n\lambda$, we get

$I = 4a^2$

i.e., intensity is maximum when the phase difference is whole number multiple of 2π or the path difference is whole number multiple of wave length.

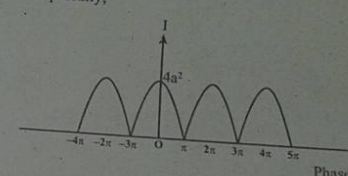
Case II

When the phase difference $\delta = \pi, 3\pi, \ldots\ldots, (2n + 1)\pi$, or the path difference $x = \frac{3\lambda}{2}, \ldots\ldots\ldots, (2n+1)\frac{\lambda}{2}$, we get

$I = 0$

Intensity is minimum when the phase difference is odd number integer multiple of π or path difference is odd number multiple of half wavelength.

Graphically,



OR,

Show that the intensity of second primary maxima is 1.62% of central maxima in Fraunhoffer's single slit diffraction.

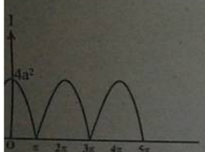
of amplitude of resultant wave.

$\delta = 0, 2\pi, 4\pi, \ldots\ldots n(2\pi)$ or path λ, 3λ,....., nλ, we get

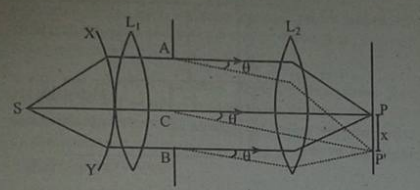
ximum when the phase difference is le of 2π or the path difference is whole vave length.

erence $\delta = \pi, 3\pi, \ldots\ldots, (2n+1)\pi$, or the $\frac{\lambda}{2}$ ...., $(2n+1)\frac{\lambda}{2}$, we get

m when the phase difference is odd ltiple of π or path difference is odd alf wavelength.



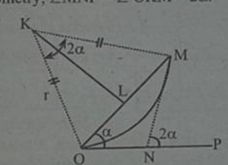
nsity of second primary maxima is axima in Fraunhoffer's single slit



Consider a single slit AB of width a. A cylindrical wave front XY is incident on slit after refraction through lens L_1. These parallel rays are collected on screen by lens L_2. To study the intensity distribution on screen, this wavefront can be imagined to be divided into large number of small strips. The resultant amplitude due to all individual strips can be obtained by the vector polygon method. Here, the amplitudes are small and phase difference increases by infinitesimally small amounts from strip to strip. Thus, vibration OP gives direction of initial vector and MN direction of final vector and k is the centre of the circular arc

Let α be the phase difference between the secondary waves from the points B and A of the slit.

From geometry, $\angle MNP = \angle OKM = 2\alpha$.



In Δ OKL, $\sin\alpha = \frac{OL}{r} \Rightarrow OL = r\sin\alpha$

where r is radius of circular arc.

Chord OM = 2 OL = 2 r sin α ....(1)

The length of arc OM is proportional to the width of the slit.

∴ Length of the arc OM = ka

where k is proportionality constant and a is slit width.

$$\text{or, } 2\alpha = \frac{\text{Arc OM}}{\text{radius}} = \frac{ka}{r}$$

$$\text{or, } 2r = \frac{ka}{\alpha} \ldots\ldots\ldots(2)$$

But OM = A is amplitude of resultant wave

$$\therefore A = ka\frac{\sin\alpha}{\alpha} = A_0\frac{\sin\alpha}{\alpha}$$

$$\text{And, intensity (I)} = A^2 = A_0^2\frac{\sin^2\alpha}{\alpha^2} = I_0\frac{\sin^2\alpha}{\alpha^2}$$

$$\text{Now, } 2\alpha = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\text{or, } \alpha = \frac{\pi}{\lambda} a\sin\theta \ldots\ldots\ldots(3)$$

$$\text{For maxima, } a\sin\theta_n = \frac{(2n+1)\lambda}{2}$$

$$\text{or, } \sin\theta_n = \frac{(2n+1)\lambda}{2a} \ldots\ldots\ldots(4)$$

From (3) & (4), we have

$$\alpha = \frac{\pi}{\lambda}\frac{a(2n+1)\lambda}{2a} = (2n+1)\frac{\pi}{2}$$

Substituting n = 2 for second primary maxima, we get

$$\alpha = \frac{5\pi}{2}$$

$$\therefore I = I_0\frac{\text{Sin}^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2}$$

$$I = 0.0162\, I_0$$

or, $I = 1.62\%$ of I_0 proved

5. **A beam of plane polarized light is converted into circularly polarized light by passing it through a crystal slice of thickness 3×10^{-5} m. Calculate the difference in the refractive indices of the two rays inside the crystal. Wavelength of light is 600nm.**

⇒ Thickness (t) = 3×10^{-5}m

Wavelength (λ) = 600 nm = 6×10^{-7}m

We know that plane polarized light is converted in to circularly polarized light by quarter wave plate.

Therefore, $t = \frac{\lambda}{4(\mu_E - \mu_0)}$

$\therefore (\mu_E - \mu_0) = \frac{\lambda}{4t} = \frac{6\times10^{-7}}{4\times3\times10^{-5}} = 0.5\times10^{-2} = 0.005$

6. **What are active medium, population inversion and optical pumping? Give the importance in the study of LASER. Write a method for getting He-Ne LASER.**

⇒ **Active medium**

It is a medium in which light gets amplified and this medium may be solid, liquid, or gas. It is important to note that only a fraction of a particular medium is responsible to stimulated emission and remaining part of the medium merely supports the active centres.

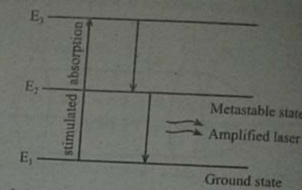
Population inversion

The establishment of a situation in which the number of atoms in the higher energy level is greater than that in the lower energy level is called population inversion.

Pumping

The procedure adopted to achieve population inversion is called pumping.

In optical pumping, active medium is illuminated by light of suitable frequency $\upsilon = \frac{E_2 - E_1}{h}$. As a result, atoms in lower energy state E_1 absorb incident photon of energy $h\upsilon$ and raised to higher energy state E_2.

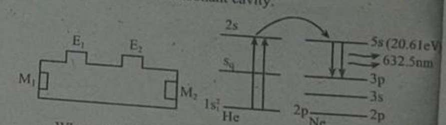


Study of laser is very important in various engineering fields. Laser is used for the following purposes.

1. Distance measurement.
2. Laser is highly intense. So, it is used in welding, of materials, etc.
3. Laser is used in automatic control of rocket and spaceship.
4. Lasers are used in eye surgery.

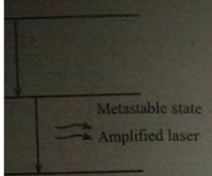
He–Ne laser

It consist of a long narrow discharge tube filled with mixture of Helium and Neon. Two electrodes E_1 and E_2 are fitted to discharge tube. M_1 & M_2 are two mirrors which forms resonant cavity.



When electric field is applied, He-atoms are excited to 2s state (20.6/eV) same as energy of 5s of Ne atom., So, energy of He–atoms are transferred to Ne–atoms.

This process results in population inversion between 5s & 3p state in Ne-atom. The spontaneous transition from 5s to 3p would produce photons of wavelength 632.5nm, which then trigger stimulated emission and photon travelling parallel to



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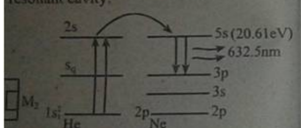
a long narrow discharge tube filled with elium and Neon. Two electrodes E_1 and E_2 discharge tube. M_1 & M_2 are two mirror resonant cavity.



ld is applied, He-atoms are excited to 2s me as energy of 5s of Ne atom., So, energy ransferred to Ne–atoms.

s in population inversion between 5s & 3p The spontaneous transition from 5s to 3p otons of wavelength 632.5nm, which then emission and photon travelling parallel to

tube are reflected back and forth between mirror and rapidly build up an intense beam which escape through the end with lower reflectivity, which is a laser beam.

7. **Write the physical meaning of dispersive power and resolving power of plane transmission grating. Show that the product of the total number of ruling and the order of the spectrum gives the resolving power of the plane transmission grating.**

⇒ ..ispersive power of a grating is defined as the change in angle of diffraction corresponding to a unit change in the wavelength i.e., dispersive power is the capacity of a grating to diffract the light wave of various wavelengths differently. Mathematically, it is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sin\theta}$$

Resolving power is the capacity of grating to resolve the light waves of very close wavelength. Mathematically, it is defined as the ratio of wavelength of light to unit change of wavelength.

For n^{th} order principal maxima for wavelength λ, for normal incidence, and a grating element (a + b)

$$(a+b)\sin\theta = n\lambda \text{ ....(1)}$$

For n^{th} maxima for wavelength $\lambda + d\lambda$

$$(a+b)\sin(\theta + d\theta) = n(\lambda + d\lambda) \text{ ....(2)}$$

Two lines will be resolved if extra path difference of $\frac{\lambda}{N}$ is introduced after n^{th} maxima.

$$(a+b)\sin(\theta + d\theta) = n\lambda + \frac{\lambda}{N} \text{ ....(3)}$$

From (2) & (3), we get

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

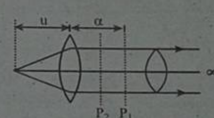
$$n\lambda + n\,d\lambda = n\lambda + \frac{\lambda}{N}$$

$$\therefore \frac{\lambda}{d\lambda} = nN$$

Hence, the product of total number of rulings and the order of spectrum gives the resolving power of transmission grating.

8. **Two thin identical convex lenses of focal length 8 cm and each are coaxial and 4 cm apart. Find the principal points and the position of object for which image is formed at infinity.**

⇒



$f_1 = 8$cm, $f_2 = 8$ cm, d = 4 cm

Focal length of equivalent lens (f) $= \frac{f_1 f_2}{f_1 + f_2 - d}$

$$= \frac{8 \times 8}{8 + 8 - 4} = 5.33 \text{ cm}$$

$$\alpha = \frac{f.d}{f_2} = \frac{5.33 \times 4}{8} = 2.66 \text{ cm}$$

$$\beta = -\frac{f.d}{f_1} = -\frac{5.33 \times 4}{8} = -2.66 \text{ cm}$$

Using the relation

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$V = \infty$, f = 5.33 cm

U = –f = –5.33 cm

Using, $U = u - \alpha$

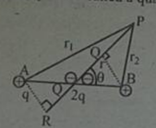
or, $u = U + \alpha = -5.33 + 2.66$

$\therefore u = -2.67$ cm

Therefore, the object is at a distance of 2.67 cm to the left of the first lens.

9. **What is electric quadrapole? Calculate the electric potential of a linear quadrapole of separation 2z at an axial distance r from its center.**

⇒ **Electric quadrapole:** The arrangement of four equal and opposite charges or two dipoles arrangement with certain electric field at any point is called a quadrapole.



Let AB be linear quadrapole of separation 2z. Let P be the point at distance r from centre O of quadrapole at which electric potential is to be calculated.

$\angle BOP = \angle AOR = \theta$, $RO = OQ = z\cos\theta$, $BQ = RA = z\sin\theta$

From figure, $r_1^2 = AR^2 + RP^2 = (z\sin\theta)^2 + (r + z\cos\theta)^2$

or, $r_1 = \sqrt{z^2 + r^2 + 2zr\cos\theta}$

Similarly, $r_2 = \sqrt{z^2 + r^2 - 2zr\cos\theta}$

Electric potential due to the quadrapole at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{z^2+r^2+2zr\cos\theta}} + \frac{1}{\sqrt{z^2+r^2-2zr\cos\theta}} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{z^2}{r^2} + \frac{2z\cos\theta}{r}\right)^{\frac{-1}{2}} + \frac{1}{r}\left(1 + \frac{z^2}{r^2} - \frac{2z\cos\theta}{r}\right)^{\frac{-1}{2}} - \frac{2}{r}\right]$$

Using binominal expansion, the expression is

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[1 - \frac{1}{2}\left(\frac{z^2}{r^2} + \frac{2z\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{z^2}{r^2} + \frac{2z\cos\theta}{r}\right)^2 + \dots\right]$$

$$+\left[\left(1 - \frac{1}{2}\left(\frac{z^2}{r^2} - \frac{2z\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{z^2}{r^2} - \frac{2z\cos\theta}{r}\right)^2 + \dots\right)\right]$$

Neglecting higher terms, we get

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[-\frac{z^2}{r^2} + \frac{6}{8}\left(\frac{2z\cos\theta}{r}\right)^2\right]$$

or, $V = \frac{qd}{4\pi\varepsilon_0 r^3}(3\cos^2\theta - 1)$

This is the expression for electric potential at any point P at an angle θ at a distance r from the centre of the quadrapole.

For axial line, we substitute $\theta = 0$.

$\therefore V = \frac{2qd^2}{4\pi\varepsilon_0 r^3} = \frac{Q}{4\pi\varepsilon_0 r^3}$

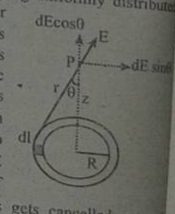
where $Q = 2qd^2$ is electric quadrapole moment due to charge assembly.

OR,

A ring radius "R" is carrying a uniformly distributed charge "q". Find an expression for electric field at any point on the axial line. Find the point at which electric field is maximum.

⇒

Consider a ring of radius R carrying uniformly distributed positive charge q with linear charge density λ. The ring is divided into elementary segments each of length dl. Let the electric field intensity dE due to this segment makes an angle θ with vertical. So, it can be resolved into two components dE sinθ and dE cos θ. If we consider the effect of whole ring, dE sinθ components gets cancelled out and resultant field is



10.

⇒

$\frac{2z\cos\theta}{r}\Big)^2 + \ldots\Big) - 2\Big]$

ntial at any point P at
e of the quadrapole.

moment due to charge

niformly distributed
electric field at any
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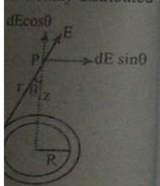


cancelled out and

$E = \int dE\cos\theta$

where $dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0(R^2+z^2)}$, $\cos\theta = \frac{z}{\sqrt{R^2+z^2}}$

or, $E = \frac{z\lambda}{4\pi\varepsilon_0(R^2+z^2)^{3/2}}\int_0^{2\pi R} dl$

or, $E = \frac{qz}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}}$

Differentiating with respect to z, we get

$$\frac{dE}{dz} = \frac{qz}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}} - \frac{3}{2}(R^2+z^2)^{\frac{-5}{2}} \times 2z\frac{qz}{4\pi\varepsilon_0}$$

$$= \frac{q}{4\pi\varepsilon_0}(R^2+z^2)^{\frac{-3}{2}}\left[1 - \frac{3}{2(R^2+z^2)} \times 2z^2\right]$$

For maximum intensity, $\frac{dE}{dz} = 0$

or, $\frac{3z^2}{z^2+R^2} = 1$

or, $3z^2 = z^2 + R^2$

or, $z^2 = \frac{R^2}{2}$

$\therefore z = \pm\frac{R}{\sqrt{2}}$

Hence, the electric field is maximum at distance $\pm\frac{R}{\sqrt{2}}$.

10. A cylindrical resistor of radius 6 mm and length 2.5 cm is made of material that has a resistivity of 4 × 10^{-5} Ω.m. What are (i) the magnitude of the current density and (ii) the potential difference when the energy dissipation rate in the resistor is 2 watt?

⇒ Radius of resistor (r) = 6 mm = 6×10^{-3} m

length (l) = 2.5 cm = 2.5×10^{-2} m

Resistivity (ρ) = $4\times10^{-5}\ \Omega$ m

i. We have $P = i^2R = J^2A^2R$

or, $J = \frac{1}{A}\sqrt{\frac{P}{R}}$

or, $J = \frac{1}{A}\sqrt{\frac{PA}{\rho l}}$ $\left[\because R = \frac{\rho l}{A}\right]$

or, $J = \sqrt{\frac{P}{\rho l A}} = \sqrt{\frac{2}{4\times10^{-5}\times2.5\times10^{-2}\times\pi(6\times10^{-3})^2}}$

$= 1.33\times10^5\ A/m^2$

$P = VI = JAV$

$\therefore V = \frac{P}{JA} = \frac{2}{\pi(6\times10^{-3})^2\times1.33\times10^5} = 0.133\ V$

11. A solenoid 2.6 m long and 1.3 cm in diameter carries a current of 9A. The magnetic field inside the solenoid is 20m T. Find the length of the wire forming the solenoid.

⇒ Length of solenoid (l) = 2.6 m

diameter (d) = 1.3 cm

Current (I) = 9A

Magnetic field (B) = 20 mT = 20×10^{-3}T

Length of wire (L) =?

Magnetic field inside solenoid (B) = $\mu_0 i n = \frac{\mu_0 i N}{l}$

or, $N = \frac{Bl}{\mu_0 I}$

Total length of wire used in making solenoid is

$$2\pi rN = \frac{2\pi r Bl}{\mu_0 i} = \frac{2\times\pi(1.3\times10^{-2})\times20\times10^{-3}\times2.6}{2.(4\pi\times10^{-7})\times9} = 187.7\ m$$

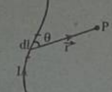
12. Compare the methods of Biot and Savart's law and Ampere's law to calculate magnetic fields due to current carrying conductor. Calculate magnetic field at an axial distance "x" from the center of the circular coil carrying current.

⇒ **Biot and Savart's law**

This method is used to find the magnetic field at any point in the region around the current carrying conductor. Any current carrying conductor can be supposed to consist of small current elements.

According to this law, magnetic field at point P is given by

$$d\vec{B} = \frac{\mu \vec{dl} \times \vec{r}}{4\pi r^3} \quad ......(1)$$



Ampere's circuital law

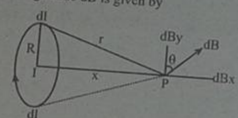
According to this law, line integral of magnetic field along closed hypothetical path in vacuum is equal to μ_0 times the current enclosed by the closed path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Calculation of magnetic field

In Biot-Savart's law, there is no closed loop around the current carrying conductor.

Consider a circular conductor with radius R, carrying current I. To find magnetic field at point P at distance x from the centre, consider an element dl of coil that sets up field dB at P. The magnitude dB is given by



$$dB = \frac{\mu_0 I dl}{4\pi(x^2 + R^2)}$$

This field has two components $dB_y = dB\cos\theta$ along y-axis and $dB_x = dB\sin\theta$ along x-axis.

If we consider the element dl' diagrammatically opposite to dl, we see that it sets up dB' equal in magnitude to dB. The y- components due to dB and dB' gets cancelled out. Thus total field B along x-axis is obtained by integrating dB sinθ all around the loop.

$$B_x = \oint dB_x = \oint dB\sin\theta = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{dl}{(x^2+R^2)} \times \frac{R}{\sqrt{x^2+R^2}}$$

$$\left[\because \sin\theta = \frac{R}{\sqrt{x^2+R^2}}\right]$$

$$\boxed{B_x = \frac{\mu_0 I R^2}{2(x^2+R^2)^{\frac{3}{2}}}}$$

This is the required expression of magnetic field due to circular loop at distance x from its centre.

13. **In a Hall experiment, a current of 25A is passed through a long foil of silver, which is 0.1 mm thick and 3 m long. Calculate the Hall voltage produces across the width by a flux of 1.4 Wb/m². If the conduction of silver is 6.8×10⁷ mho/m, estimate the mobility of the electrons in silver.**

⇒ Current (I) = 25 A

Thickness (t) = 0.1 mm = 0.1×10^{-3} m

Magnetic field (B) = 1.4 Wb/m²

Conductivity of silver (σ) = 6.8×10^7 mho/m

Hall voltage (V_H) = ?

Mobility of electron in silver (μ_e) = ?

Electron density (n) = 7.4×10^{28}/m³

Hall voltage, $V_H = \frac{Bi}{net} = \frac{1.4\times25}{7.4\times10^{28}\times1.6\times10^{-19}\times10^{-4}}$

$= 2.956\times10^{-5}$ V

Hall coeff

Mobility(μ

14. **A parallel ... by current ... in terms o... the center ... magnetic ... current?**

⇒ According t

$\oint \vec{B} \cdot d\vec{s}$ ...

$A = \pi\left(\frac{a}{5}\right)^2$ ...

$I_{enclosed} = I \times$ ...

$\oint \vec{B} \cdot d\vec{s} =$ ...

For induced ...

$\oint \vec{B} \cdot d\vec{s}$ ...

$B(2\pi\frac{a}{5})$ ...

$\therefore B = \frac{\mu_0 i d}{10\pi a}$

OR,

An inductance ... through a resi...

Hall coefficient $(R_H) = -\frac{1}{ne}$

$$= \frac{1}{7.4\times10^{28}\times1.6\times10^{-19}}$$

$$= -8.4\times10^{-11}\ m^3/C$$

$$\text{Mobility}(\mu_e) = R_H\,\sigma = \frac{R_H}{\rho} = 8.4\times10^{-11}\times6.8\times10^{7}$$

$$= 57.43\times10^{-4}\ m^2V^{-1}s^{-1}$$

14. A parallel plate capacitor with circular plates is charged by current "i". (a) What is the magnitude of $\oint \vec{B}.d\vec{s}$ in terms of μ_0 and i between the plates if r = (a/5) from the center? (b) What is the magnitude of induced magnetic field for r = (a/5) in terms of displacement current?

⇒ According to Ampere's circuital law

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enclosed}$$

$$A = \pi\left(\frac{a}{5}\right)^2,\ J = \frac{I}{A} = \frac{I}{\pi a^2}$$

$$I_{enclosed} = J\times A = \frac{I}{\pi a^2}\times\pi\left(\frac{a}{5}\right)^2 = \frac{i}{25}$$

$$\oint \vec{B}.d\vec{s} = \frac{\mu_0 i}{25}$$

For induced magnetic field

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enclosed}$$

$$B\left(2\pi\frac{a}{5}\right) = \frac{\mu_0 i}{25}$$

$$\boxed{\therefore B = \frac{\mu_0 i_d}{10\pi a}}$$

OR,

An inductance L is connected to a battery of emf E through a resistance. Show that the potential difference across the inductance after time t is $V_L = \varepsilon e^{\left(\frac{-R}{L}\right)t}$. At what time is the potential difference across the inductance equal to that across the resistance such that $i = \frac{i_0}{2}$.

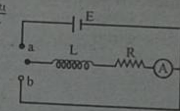
⇒ The growth of the current through RL circuit is

$$i = i_0\left(1 - e^{-t/\tau}\right) = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

Potential difference across the inductor is

$$V_L = L\frac{di}{dt} = -L\,i_0\left(\frac{-R}{L}\right)e^{\frac{-Rt}{L}}$$

$$= i_0 R e^{\frac{-Rt}{L}}$$



$\boxed{V_L = \varepsilon e^{\frac{-Rt}{L}}}$ proved

When the potential difference across the inductor is equal to that of resistance, we write

$$V_L = V_R$$

$$\text{or, } \varepsilon e^{\frac{-Rt}{L}} = IR$$

$$\therefore e^{\frac{Rt}{L}} = \frac{\varepsilon}{iR}$$

According to question, $i = \frac{i_0}{2}$

$$\text{or, } \frac{i_0}{i} = 2$$

$$\text{or, } e^{\frac{Rt}{L}} = 2$$

$$\text{or, } \frac{Rt}{L} = \ln 2$$

$$\text{or, } t = \frac{L}{R} \ln 2$$

$$\therefore t = 0.693\frac{L}{R} \text{ sec}$$

15. **Write Maxwell equations in integral form. Convert them in differential form. Explain the physical meaning of each equation.**

⇒ *Refer to Q.N. 15 of 070 Ashad*

16. **Describe the physical significance of the wave function. Derive an expression of time dependent Schrödinger equation.**

⇒ **Physical significance of wave function Ψ**

In classical mechanics, the wave is represented by equation

$$y = A\, e^{\frac{-i}{\hbar}(Et-px)} \quad ....(1)$$

This function in general is a complex quantity and is dependent upon space and time and is denoted by

$$\Psi(x, t) = A\, e^{\frac{-i}{\hbar}(Et-px)} \quad ....(2)$$

This function has no physical significance itself. The only quantity having physical meaning is the square of its magnitude.

i.e., $P = \Psi\Psi^* = |\Psi|^2$.

This is called probability density, where Ψ^* is complex conjugate of Ψ. The probability of finding particle in the volume element dv can be expressed as $|\Psi|^2$ dv and the probability of finding particle in entire space is $\int_{-\infty}^{\infty} |\Psi|^2\, dv = 1$.

Differentiate equation (2) with respect to time, we get

$$\frac{d\Psi}{dt} = A\left(\frac{-i}{\hbar}\right) E\, e^{\frac{-i}{\hbar}(Et-px)} = -i\frac{E}{\hbar}\Psi$$

$$\text{or, } i\hbar \frac{d\Psi}{dt} = E\,\Psi \quad ....(3)$$

Difference $\Psi(x, t)$ w.r. to x, we have

$$\frac{d\Psi}{dx} = A\frac{ip}{\hbar}\, e^{\frac{-i}{\hbar}(Et-px)} \quad ....(4)$$

Differentiating again will result

$$\frac{d^2\Psi}{dx^2} = A\left(\frac{-i}{\hbar}\right)^2 e^{\frac{-i}{\hbar}(Et-px)}$$

$$\text{or, } \frac{d^2\psi}{dx^2} = \frac{-p^2}{\hbar^2}\Psi. \quad ....(5)$$

$$\text{Total energy, } E = K + V = \frac{p^2}{2m} + V$$

$$\text{or, } p^2 = 2m(E-V)$$

Operating this operator on Ψ, we get

$$p^2\,\Psi = 2m(E-V)\Psi$$

From equation (5), $-\hbar^2\frac{d^2\Psi}{dx^2} = 2m(E-V)\Psi \quad ....(6)$

Using equation (3) in (6), we get

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\,\Psi = i\hbar\frac{d\Psi}{dt}$$

In three dimension,

$$\boxed{i\hbar\frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi}$$

This is the required Schrödinger time dependent wave equation.

$\hbar \frac{d\Psi}{dt} = E\,\Psi$ ....(3)

ence Ψ (x, t) w.r. to x, we have

$\frac{\Psi}{x} = A \frac{i\,p}{\hbar} e^{\frac{-i}{\hbar}(Et-px)}$ ....(4)

entiating again will result

$= A\left(\frac{-i}{\hbar}\right)^2 e^{\frac{-i}{\hbar}(Et-px)}$

$= \frac{-p^2}{\hbar^2}\,\Psi$ ....(5)

nergy, $E = K + V = \frac{p^2}{2m} + V$

or, $p^2 = 2m\,(E-V)$

ing this operator on Ψ, we get

$\Psi = 2m(E-V)\Psi$

quation (5), $-\hbar^2 \frac{d^2\Psi}{dx^2} = 2m\,(E-V)\Psi$ ....(6)

equation (3) in (6), we get

$\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\,\Psi = i\,\hbar \frac{d\Psi}{dt}$

e dimension,

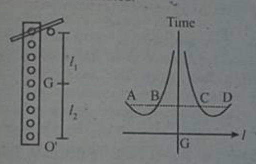
$\boxed{\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi}$

the required Schrödinger time dependent wave n.

2070 Bhadra

1. **Derive a relation to determine the radius of gyration of a compound pendulum. Why is determination of the acceleration due to gravity more accurate from a compound pendulum than a simple pendulum?**

⇒ A compound pendulum consists of a metal bar with large number of holes as shown in figure. Radius of gyration is defined as the perpendicular distance of the axis of rotation from a point at which the total mass of the body may be assumed to be concentrated.



Time period can be measured by fixing the knife edge in each corresponding holes. The distance of each hole from the centre of gravity is measured. If a graph between distance from centre of gravity and corresponding time period is plotted, it looks like as shown in fig.

Two points not equal from centre of gravity G in bar pendulum O and O' on both sides of it, one corresponding to centre of suspension and other as centre of oscillation. Let their distance be ℓ_1 and ℓ_2 from the centre of gravity as shown. Thus, the equivalent length of the pendulum is $L = L_1 + L_2$.

The time period of compound pendulum is given by

$$T = 2\pi\sqrt{\frac{k^2 + l_1^2}{l_1 g}} = 2\pi\sqrt{\frac{\frac{k^2}{l_1} + l_1}{g}} \text{ ......(1)}$$

Comparing equation (i) with $T = 2\pi\sqrt{\frac{L}{g}}$ .....(2), we get

$$L = \frac{k^2}{l_1} + l_1$$

$$\text{or, } l_1 + l_2 = \frac{k^2}{l_1} + l_1$$

$$\text{or, } k^2 = l_1 l_2$$

$$\text{or, } k = \sqrt{l_1 l_2}$$

In this way, the value of radius of gyration k about the axis passing through the centre of gravity of a bar pendulum can be determined.

Determination of acceleration due to gravity from compound pendulum is more accurate due to following reasons:

i. It is not an ideal concept i.e., it can be realized in actual practice.

ii. There is no lag like that in between bob and string in case of simple pendulum.

iii. Distance between two knife edge can be easily measured but it is difficult to measure the accurate length of simple pendulum.

iv. Due to larger mass and hence, larger moment of inertia, it continues to oscillate for a longer time

OR,

Define the quality factor (Q). Derive a relation of quality factor (Q) from the damped harmonic motion and show that the quality factor (Q) is inversely proportional to damping constant (b).

⇒ **Quality factor (Q)**

Quality factor of a damped oscillator defines the quality of oscillator so far as damping is concerned. Less is the damping, higher is the quality factor Q. Mathematically, it is defined as

$$Q = 2\pi \times \frac{\text{Energy stored}}{\text{Energy lost per cycle}}$$

If ω_1 and ω_2 be the frequencies at which the amplitude becomes $\frac{A_0}{\sqrt{2}}$, then the quantity $\omega_1 - \omega_2$ is called width of resonance peak. Thus, quality is related with $\omega_1 - \omega_2$ by

$$\frac{\Delta\omega}{\omega} = \frac{1}{Q} \text{ ....(1)}$$

The energy per cycle of damped harmonic motion is

$$E_{avg} = \frac{1}{2} m\omega^2 x_m^2 e^{\frac{-bt}{m}} \text{ ....(2)}$$

where b = damping constant

As $\frac{b}{m} = 2\delta = \frac{1}{\tau}$, 2δ is logarithmic decrement and τ is relaxation time. Then, above equation becomes

$$E_{avg} = \left[\frac{1}{2} m\omega^2 x_m^2\right] e^{-2\delta t}$$

$$= E_0 e^{-2\delta t} \text{ .....(3)}$$

Average power dissipation is

$$P = -\frac{dE_{avg}}{dt} = -\frac{d}{dt}(E_0 e^{-2\delta t}) = 2\delta E_0 e^{-2\delta t}$$

$$\text{or, } P = 2\delta E_{avg} \text{ ....(4)}$$

$$\therefore \frac{E_{avg}}{P} = \frac{1}{2\delta} \text{ ....(5)}$$

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy loss per period}}$$

$$\text{or, } Q = 2\pi \frac{E_{avg}}{PT}$$

$$\text{or, } Q = \omega \frac{E_{avg}}{P}$$

$$\text{or, } Q = \frac{\omega}{2\delta}$$

$$\text{or, } Q = \omega\tau.$$

$$\boxed{\therefore Q = \frac{m\omega}{b}} \text{ ....(6)} \quad \left[\text{since } \tau = \frac{b}{m}\right]$$

This is the expression for quality factor of damped harmonic oscillator. From equation (6), it is clear that quality factor is inversely proportional to damping constant b.

2. **An oscillatory motion of a body is represented by $y = a e^{i\omega t}$ where y is displacement in time t, a is its amplitude and ω is angular frequency. Show that the motion is simple harmonic.**

⇒ The displacement of a body executing simple harmonic motion is given by

$$y = a e^{i\omega t} \text{ ....(1)}$$

Differentiating w.r.t. time will give

$$\frac{dy}{dt} = (i\omega) a e^{i\omega t} \text{ ....(2)}$$

Differentiating equation (2) will result

$$\frac{d^2y}{dt^2} = (i\omega)^2 a e^{i\omega t}$$

$$\text{or, } \frac{d^2y}{dt^2} = -\omega^2 y$$

$$\text{or, } \frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ ....(3)}$$

which is second order differential equation of simple harmonic motion. Hence, the motion represented by equation (1) is simple harmonic.

3. **What is ultrasound? How these waves are produced? Differentiate such waves from ordinary sound wave.**

⇒ **Ultrasound**

The sound wave which has frequency greater than 20,000 Hz is called ultrasonic sound wave. Since they travel with velocity of sound, marginal wavelength of such wave is $\frac{33200}{20000} = 1.66$ cm at room temperature.

Production of ultrasonic wave- Piezo–electric generation

Certain crystals like Quartz, Tourmaline, etc. exhibit electrical charges when heated or cooled. This phenomenon is called piezoelectricity.

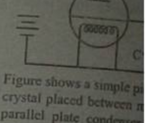


Figure shows a simple pi
crystal placed between n
parallel plate condenser
primary of a transformer
oscillator circuit of a va
frequency of oscillation
frequency of the crystal n
resonance will occur a
expansion and contractio
moderate size crystal, ult
cycles/sec can be pro
longitudinal in nature and f

$$f = \frac{k}{2l}\sqrt{\frac{Y}{\rho}}$$

where k = 1, 2, 3, ...etc. is

Y = Young's modulus

ρ = density of the crys

Ultra sound
1. Frequency of ultrasound is greater than 20,000 Hz
2. It can't be heard.
3. It has many applications in engineering.

...ory motion of a body is represented by $y = a^{i\omega t}$ ... displacement in time t, a is its amplitude and ω ... frequency. Show that the motion is simple

...acement of a body executing simple harmonic ... given by

$e^{i\omega t}$..........(1)

...ating w.r.t. time will give

$(i\omega)\, a\, e^{i\omega t}$ ....(2)

...iating equation (2) will result

$= (i\omega)^2\, a\, e^{i\omega t}$

$\frac{d^2y}{dt^2} = -\omega^2 y$

$\frac{d^2y}{dt^2} + \omega^2 y = 0$.........(3)

...s second order differential equation of simple ...c motion. Hence, the motion represented by ... (1) is simple harmonic.

...s ultrasound? How these waves are produced? ...tiate such waves from ordinary sound wave.

...und

...nd wave which has frequency greater than 20,000 Hz ...d ultrasonic sound wave. Since they travel with ... of sound, marginal wavelength of such wave is ... 1.66 cm at room temperature.

...on of ultrasonic wave- Piezo–electric generation

... crystals like Quartz, Tourmaline, etc. exhibit ... charges when heated or cooled. This phenomenon ... piezoelectricity.

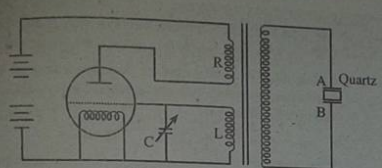


Figure shows a simple piezoelectric generator. Q is a slice of crystal placed between metal plates A and B so as to form parallel plate condenser. The plates are connected to the primary of a transformer which is coupled inductively to the oscillator circuit of a valve as shown in figure. If natural frequency of oscillation of valve circuit is equal to the frequency of the crystal mode to which we intend to excite, resonance will occur and crystal will undergo linear expansion and contraction at exactly same rate. With moderate size crystal, ultrasonics of frequency 5, 40,000 cycles/sec can be produced. These vibrations are longitudinal in nature and frequency of vibration is given by

$$f = \frac{k}{2l}\sqrt{\frac{Y}{\rho}}$$

where k = 1, 2, 3,......etc. is mode of vibration

Y = Young's modulus of elasticity

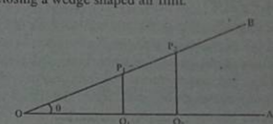
ρ = density of the crystal

Ultra sound	Ordinary sound
1. Frequency of ultrasound is greater than 20,000 Hz	1. Frequency of ordinary sound ranges from 20 Hz to 20,000 Hz.
2. It can't be heard.	2. It can be heard.
3. It has many applications in engineering.	3. It has limited applications.

4. **Why are colours observed when soap bubble is exposed to sunlight? Show that the consecutive bright or dark fringes are observed when the thickness of the film increases $\frac{\lambda}{2}$ in an inclined plane.**

⇒ A ray from sunlight consists of seven different colours. When it is incident on thin film of soap bubble, it reflects from upper layer and lower layers of the bubbles. The reflected rays from two layers behave as coherent rays and interfere with one another. The path difference between these two reflected rays are different for different colours. So, coloured fringes will observed due to interference effect.

Consider a plane surface OA and OB inclined at an angle θ enclosing a wedge shaped air film.



Suppose n^{th} bright fringe occurs at P_1. Thickness of air film at P_1 is P_1Q_1. Applying the relation for bright fringe, we write

$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ ....(1)

For air $\mu = 1$, for small angle r, $\cos r \approx 1$.

or, $2t = (2n+1)\frac{\lambda}{2}$

or, $2P_1Q_1 = (2n+1)\frac{\lambda}{2}$ ....(2)

Next bright fringe (n+1) will occur at P_2 such that

$2P_2Q_2 = [2(n+1) + 1]\frac{\lambda}{2}$

$$\text{or, } 2P_2Q_2 = (2n+3)\frac{\lambda}{2} \text{ ....(3)}$$

Subtracting (2) from (3), we get

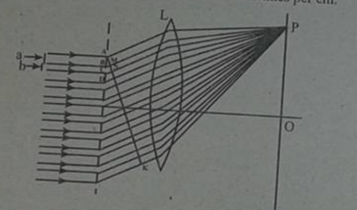
$$P_2Q_2 - P_1Q_1 = \frac{\lambda}{2}$$

Thus, the next bright fringe will occur at the point where the thickness of the air film increases by $\frac{\lambda}{2}$. Similarly, for dark fringe, the above relation holds true.

OR,

What is plane diffraction grating? How is it used to find the wavelength of a monochromatic light experimentally?

⇒ A plane diffraction grating consists of an optically plane glass plate on which are ruled a number of equidistance parallel straight lines. The lines divide the glass plate into opacities and transparencies, the thickness of which is of the order of wavelength of visible light. The region where a line is drawn becomes opaque whereas the space between the two lines is transparent. The number of lines in a plane transmission grating is of the order of 6000 lines per cm.



Let the width of transparency AB = a and width of opacity BC = b. The distance (a + b) is called grating element.

The path difference between the rays from A and from C is CN = (a + b) $\sin\theta_n$, where θ is called the angle of diffraction. The point P will be bright or dark according as the rays reinforce or interfere with one another. They would reinforce and give brightness if

$$(a + b)\sin\theta_n = n\lambda \text{ .....(1)}$$

They would interfere and produce darkness or minima if

$$(a + b)\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

To find the wavelength of monochromatic light by plane diffraction grating, angle of diffraction is measured for first order and second order. Then, by using above relation, value of λ can be determined.

For first order diffraction, $(a + b)\sin\theta_1 = \lambda$

For second order diffraction $(a + b)\sin\theta_2 = 2\lambda$

5. **What is an optical fiber? How is it made? Write down the main differences between step index and graded index multimode optical fibers with well diagrams.**

⇒ It is a flexible optically transparent fiber, usually made of glass or plastic, through which light can be transmitted by successive total internal reflections. Optical fiber consists of three parts namely protective layer, cladding, and core. The refractive index of the core is greater than that of the cladding. Optical fibers are fabricated from glass or plastic polymer.

It is made by arranging concentric cylinders of different refractive index. The innermost cylinder known as core is made of material of very high refractive index due to the fact that it is highly doped. The cylinder outside the core is known as cladding whose refractive index is less than that of core because this region is lightly doped. The next layer is known as buffer coating (also known as protective layer) as shown in figure.

Step inde

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of claddin
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Graded ind

In this fiber,
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surface as sh

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self focusing.

... from A and from C is
θ is called the angle of diffraction.
...ght or dark according as the rays
... one another. They would reinforce

...(1)

... produce darkness or minima if

... of monochromatic light by plane
... of diffraction is measured for first
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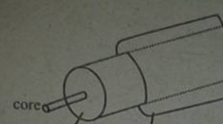
$(a + b) \sin\theta_1 = \lambda$

...n $(a + b) \sin\theta_2 = 2\lambda$

...? How is it made? Write down ...etween step index and graded ...fibers with well diagrams.

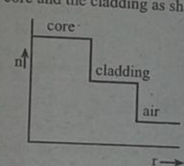
...ransparent fiber, usually made of ...which light can be transmitted by ...flections. Optical fiber consists of ...ive layer, cladding, and core. The ...core is greater than that of the ...e fabricated from glass or plastic

...concentric cylinders of different ...most cylinder known as core is ...h refractive index due to the fact ...e cylinder outside the core is ...ractive index is less than that of ...ghtly doped. The next layer is ... known as protective layer) as



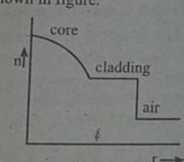
Step index optical fiber

In this fiber, the refractive index of the core is fixed and that of cladding is also fixed. Refractive index of core is higher than cladding. Therefore, there is noticeable boundary between the core and the cladding as shown in figure.



Graded index optical fiber

In this fiber, refractive index of core is a function of radial distance from the centre of the fiber i.e., refractive index of fiber decreases smoothly from the middle to the outer surface as shown in figure.



In graded index fiber, there is low transmission loss due to self focusing.

6. **A 200 mm long glass tube is filled with a solution of sugar, containing 15 gm of sugar in. 100 ml of water. The plane of polarized light, passing through this solution, is rotated through 25°17′. Find the specific rotation of sugar.**

⇒ Length of tube (L) = 200 mm = 20 cm

Concentration of solution (C) = 15 gm/cc

Angle of rotation $(\theta) = 25°\,17' = \left(25 + \frac{17}{60}\right)^{\circ} = 25.28°$

Specific rotation (S) = ?

$$S = \frac{10\theta}{L.C} = \frac{10 \times 25.28}{20 \times 15} = 84.2°$$

7. **Two thin converging lenses of focal lengths 0.2 m and 0.3 m are placed coaxially 0.1 m apart in air. An object is located 0.6 m in front of the lens of smaller focal length. Find the position of principal points and that of image.**

⇒ $f_1 = 0.2$ m, $f_2 = 0.3$ m, d = 0.1 m

Object distance (u) = 0.6 m

Focal length of equivalent lens $(f) = \frac{f_1 . f_2}{f_1 + f_2 - d}$

$$\therefore \quad f = \frac{0.2 \times 0.3}{0.2 + 0.3 - 0.1} = 0.15 \text{ m}$$

$$\alpha = \frac{f.d}{f_2} = \frac{0.15 \times 0.1}{0.3} = 0.05 \text{ m}$$

$$\beta = -\frac{f.d}{f_1} = -\frac{0.15 \times 0.1}{0.2} = -0.075 \text{ m}$$

Object distance for equivalent lens U = −(0.6+0.05)

= − 0.65 m

We have, $\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$

or, $\frac{1}{0.15} = \frac{1}{V} + \frac{1}{0.65}$

$\therefore \quad V = \frac{0.15 \times 0.65}{0.65 - 0.15} = 0.195 \text{ m}$

Now, position of image from second lens is

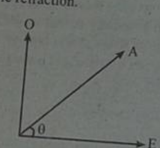
$$v = V + \beta = (0.195 - 0.075) = 0.12 \text{ m}$$

The final image is formed at a distance of 0.12 m to the right of the second lens.

8. **What is double refraction? Show that a beam of plane polarized light is converted into elliptically polarized light when it passes through a quarter-wave plate.**

⇒ **Double refraction**

When an ordinary unpolarized light is incident on calcite crystal or quartz crystal, the crystal splits the refracted rays into ordinary and extra-ordinary rays. This phenomenon is called double refraction.



Suppose amplitude of incident plane polarized light on crystal is A and it makes an angle θ with optic axis. Its components along x–axis and y–axis are $A\cos\theta$ and $A\sin\theta$ respectively. Since phase difference δ is introduced between the two rays after passing through a thickness d of the crystal, we can write

For extra-ordinary ray, $x = A\cos\theta \sin(\omega t + \delta)$ .....(1)

For ordinary ray, $y = A\sin\theta \sin \omega t$ ....(2)

Let, $a = A\cos\theta, \; b = A\sin\theta$

Then, $x = a\sin(\omega t + \delta)$ ..(3)

$y = b\sin \omega t$ ......(4)

From equation (4),

$$\frac{y}{b} = \sin \omega t$$

$$\therefore \cos \omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

From equation (3),

$$\frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta$$

$$\text{or, } \frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

$$\text{or, } \frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

Squaring ,

$$\frac{x^2}{a^2} - \frac{2xy\cos\delta}{ab} + \frac{y^2}{b^2}\cos^2\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$

$$\text{or, } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\delta}{ab} = \sin^2\delta \text{ ....(5)}$$

This is the general equation of ellipse. Hence, the plane polarized light is converted into elliptically polarized light when it passes through a quarter wave plate.

9. **Obtain an expression for electric field at an axial distance x from the centre of the flat circular disc of radius R that carries a uniform surface charge density σ. Extend your result to calculate potential at a distance x.**

⇒ Consider a disc of radius R has uniform surface charge density σ. To find the electric field at point P at distance x from the centre of the disc, we consider the disc as a set of concentric rings and calculate electric field at P due to one ring. The vector sum of all those fields gives the field due to whole disc.

Let radius of one ring be y and width dy. So, charge on the ring is $dq = 2\pi y \, dy \, \sigma$ .....(1)

Field due to ring is

$\sin^2\delta$

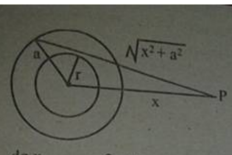
5)

...se. Hence, the plane ...tically polarized light ...plate.

...field at an axial ...lat circular disc of ...ce charge density σ. ...ial at a distance x.

...orm surface charge ... point P at distance ... the disc as a set of ...eld at P due to one ...ves the field due to

So, charge on the



$$dE = \frac{dq.x}{4\pi\varepsilon_0(x^2+y^2)^{\frac{3}{2}}} = \frac{2\pi y dy.\ \sigma x}{4\pi\ \varepsilon_0(x^2+y^2)^{\frac{3}{2}}} \quad ...(3)$$

To obtain the total field at P, integrate above expression over the limit 0 to R.

$$E = \frac{2\pi x\sigma}{4\pi\varepsilon_0}\int_0^R \frac{ydy}{(x^2+y^2)^{\frac{3}{2}}} = \frac{2\pi\ \sigma}{4\pi\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

$$\boxed{\therefore E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

which is the required expression for electric field due to disc. For one dimension, potential is

$$V = -\int E\,dx$$

$$= -\frac{\sigma}{2\varepsilon_0}\int_0^R \left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\ \int_0^R dx + \int_0^R \frac{xdx}{\sqrt{x^2+R^2}}\ \right]$$

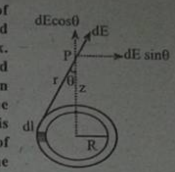
As $\int \frac{xdx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$

$$\boxed{\therefore V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+R^2} - R\right]}$$

which is the required expression for potential at distance x from a disc.

OR,

Thin ring made of plastic of radius R is uniformly charged with linear charge density x. Calculate the electric field intensity at any point at an axial distance Y from the centre. If electron is constrained to be in axial line of the same ring, show that the motion of electron is simple harmonic.



⇒ Consider a ring of radius R with uniform linear charge density λ. The ring is divided into elementary segments of length dl this segment produce electric field dE at P. Its component along x–axis will cancel out being equal and opposite. So vertical component only observed.

Total field is $E = \int dE \cos\theta$

Here, $dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda d\ell}{4\pi\varepsilon_0(R^2+y^2)}$, and $\cos\theta = \frac{y}{\sqrt{R^2+y^2}}$

or, $E = \frac{y\lambda}{4\pi\varepsilon_0(R^2+y^2)^{\frac{3}{2}}}\int_0^{2\pi R} dl$ $\qquad [\because q = 2\pi R\lambda]$

$$\boxed{\therefore E = \frac{qy}{4\pi\varepsilon_0}(R^2+y^2)^{\frac{-3}{2}}}$$

This is the required expression for electric field.

Let m be the mass of an electric and –q' be its charge. Then, force experienced by electric in above field is

$$F = -q'E = -\frac{qq'}{4\pi\varepsilon_0(R^2+y^2)^{\frac{3}{2}}}y \quad ....(3)$$

$$\text{or, } m\frac{d^2y}{dt^2} + \frac{qq'}{4\pi\varepsilon_0 (R^2+y^2)^{\frac{3}{2}}}y = 0$$

This is the differential equation of simple harmonic motion with frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{qq'}{4\pi\varepsilon_0(R^2+y^2)^{\frac{3}{2}}}}$$

Hence, the electron executes simple harmonic motion along the axis of the ring.

10. **A copper strip 2.5 cm wide and 1.5 mm thick is placed in magnetic field with 2.5 T perpendicular to the plane of the strip and away from the reader. If a current of 250 A is set up in the strip, what Hall potential difference appears across the strip? Charge density is copper = 8.4 × 10^{28}/m^3.**

⇒ Thickness of copper (t) = 1.5 mm = 1.5×10^{-3} m

Magnetic field (B) = 2.5 T

Current (I) = 250 A.

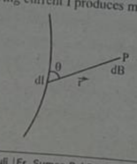
Charge density (n) = $8.4 \times 10^{28}/m^3$

We have, $V_H = \frac{BI}{net} = \frac{2.5\times250}{8.4\times10^{28}\times1.6\times10^{-19}\times1.5\times10^{-3}} = 31\ \mu V$

11. **Compare Ampere's law with Biot-Savart's law. Obtain expressions for magnetic field intensity inside and outside the long straight wire carrying current.**

⇒ **Biot-Savart's law**

A wire carrying current I produces magnetic field near it.



Consider a wire carrying current I. We want to find magnetic field at point P. Let θ be the angle between radius vector and line element dl. The magnetic field due to small current element is given by

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2} \quad \text{.........(1)}$$

Total magnetic field produced by whole length is

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{r}}{r^3} \quad \text{.........(2)}$$

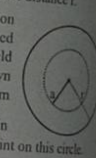
Ampere's law

If a continuous closed line is drawn round one or more current carrying conductor and B is the flux density in the direction of line element dl, then for free space, line integral of the magnetic field around loop is equal to μ_0 times the total current.

i. e., $\vec{B}\cdot d\vec{l} = \mu_0 I$.

Consider a straight wire with radius a, we can use Ampere's law to find magnetic field as a function of distance r.

i. Outside: Figure shows cross section of cylinder carrying current I directed out of the page. To find magnetic field for region r>a, amperian loop is drawn that encloses conductor as shown. From symmetriy, $\vec{B}$ must be constant in magnitude and parallel to dl at every point on this circle.



Ampere's law gives $\int\vec{B}\cdot d\vec{l} = B.\ 2\pi r = \mu_0 i$

$$\therefore B = \frac{\mu_0 i}{2\pi r} \quad \text{.......(1)}$$

ii. Inside (r < a)

Draw amperian ... shown in figure. ... enclosed by the loo...

$$\int\vec{B}\cdot d\vec{l} = \mu_0 i' ...$$

We know that curr... as

$$J = \frac{i}{\pi a^2} = \frac{i'}{\pi r^2} \Rightarrow i' ...$$

$$\therefore B(2\pi r) = \mu_0\left(\frac{r}{a}\right)^2 ...$$

$$\therefore B = \left(\frac{\mu_0 i}{2\pi a^2}\right)r \ ...$$

From equation (1) & ... & maximum at surf...

12. **A spherical drop o... a potential of 500 V... (a) What is the rad... of the same charge... spherical drop, wh... new drop?**

⇒

a. Potential (V) = $\frac{q}{4\pi\varepsilon_0 ...}$

or, $R = \frac{q}{4\pi\varepsilon_0 V} = \frac{...}{4\pi\times ...}$

c. If V be the potential...

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

If V ' be the potenti...

...agnetic field due to small current ... radius vector and ...

.....(1)

...uced by whole length is

.........(2)

...e is drawn round one or more
...and B is the flux density in the
...then for free space, line integral
...d loop is equal to μ_0 times the

...radius a, we can use Ampere's
...function of distance r.

...ss section
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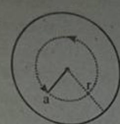
...is drawn
...wn. From

...stant in
...very point on this circle.

$= B.\,2\pi r = \mu_0 i$

ii. Inside $(r < a)$

Draw amperian loop of radius r as shown in figure. Let i' be the current enclosed by the loop, then



$$\int \vec{B}.d\vec{l} = \mu_0 i'.$$

We know that current density J is constant and is expressed as

$$J = \frac{i}{\pi a^2} = \frac{i'}{\pi r^2} \Rightarrow i' = \left(\frac{r}{a}\right)^2 i$$

$$\therefore B(2\pi r) = \mu_0 \left(\frac{r}{a}\right)^2 i$$

$$\boxed{\therefore B = \left(\frac{\mu_0 i}{2\pi a^2}\right) r} \quad ....(2)$$

From equation (1) & (2), we see that B is zero at the centre & maximum at surface.

12. **A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with V = 0 at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?**

⇒

a. Potential $(V) = \frac{q}{4\pi\varepsilon_0 R}$

or, $R = \frac{q}{4\pi\varepsilon_0 V} = \frac{30\times10^{-12}}{4\pi\times8.85\times10^{-12}\times500} = 0.54$ mm

c. If V be the potential at the surface of small drop, then

$$V = \frac{q}{4\pi\varepsilon_0 R} \quad ....(1)$$

If V ' be the potential at the surface of new drop, then

$$V' = \frac{Q}{4\pi\varepsilon_0 R'} \quad ....(2)$$

Dividing equation (2) by (1), we get

$$\frac{V'}{V} = \frac{Q}{R'} \times \frac{R}{q}$$

$$= \frac{2q}{q} \times \frac{R}{R'}$$

$$= \frac{2R}{R'} \quad ....(3)$$

Since the volume of new drop equals the volume of small drop, we can write

$$\frac{4}{3}\pi R^3 \times 2 = \frac{4}{3}\pi R'^3$$

$$\therefore R' = 2^{1/3} R$$

From equation (3), we get

$$V' = 2^{2/3} V = 2^{2/3} \times 500 = 793.7 \text{ V}$$

13. **Calculate the displacement current between the capacitor plates of area 1.5×10^{-2} m^2 and rate of electric field change is 1.5×10^{12} V/ms. Also find the value of displacement current.**

⇒ Area of plate $(A) = 1.5\times10^{-2} m^2$

Rate of electric field change $\left(\frac{dE}{dt}\right) = 1.5\times10^{12}$ V/ms

Displacement current $(i) = \varepsilon_0 A\frac{dE}{dt}$

$$= 8.85 \times 10^{-12} \times 1.5 \times 10^{-2} \times 1.5 \times 10^{12}$$

$$= 199 \text{ mA}$$

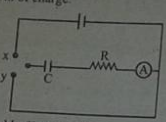
Displacement current density $(J_D) = \varepsilon_0 \frac{dE}{dt}$

$$= 8.85 \times 10^{-12} \times 1.5\times10^{12}$$

$$= 13.28 \text{A/m}^2$$

14. **Obtain expressions for growth and decay of charges in the RC circuits. Explain how you will measure experimentally the capacitance of the given capacitor.**

⇒ Consider a DC circuit containing R and C in series. When the switch x is closed, capacitor starts charging and hence, it causes growth of charge.



Applying Kirchhoff's loop rule

$$E = V_C + V_R = \frac{q}{C} + IR$$

or, $R\frac{dq}{dt} + \frac{q}{C} = E$ [since $i = \frac{dq}{dt}$]

or, $\frac{dq}{EC - q} = \frac{dt}{RC}$

Integrating, we get

$$\int \frac{dq}{q_0 - q} = \int \frac{1}{RC}\,dt \qquad [\text{since } q_0 = EC]$$

or, $-\log(q_0 - q) = \frac{t}{RC} + K$

where K is integrating constant.

For $t = 0, q = 0,$

$k = -\log q_0$

or, $-\log(q_0 - q) + \log q_0 = \frac{t}{CR}$

or, $\log\frac{q_0 - q}{q_0} = -\frac{t}{CR}$

or, $\boxed{q = q_0(1 - e^{-t/CR})}$

This is the expression for growth of charge

Decay of charge

When the capacitor is fully charged and the key ... discharging occurs through resistor.

Again, using Kirchhoff's rule

$$0 = V_C + V_R = IR + \frac{q}{C}$$

or, $R\frac{dq}{dt} = -\frac{q}{C}$

or, $\frac{dq}{q} = -\frac{1}{RC}\,dt$

Integrating, $\log q = -\frac{t}{RC} + K$

where $K = \log q_0$ is integrating constant

or, $\log q - \log q_0 = -\frac{t}{RC}$

$\boxed{q = q_0 e^{-t/RC}}$

This is the required expression for decay of charge

To find the capacitance of capacitor differentiate ... equation with respect to time

$$\frac{dq}{dt} = \frac{q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC}$$

i.e., $I = I_0\, e^{-t/RC}$

If $t_{1/2}$ is the time for current reduced to half of ... then

$$\frac{I_0}{2} = I_0 e^{\frac{t_{1/2}}{RC}}$$

Taking log on both sides, we get

or, $\frac{t_{1/2}}{RC} = \log 2$

or, $C = \frac{t_{1/2}}{R\log}$ ...

Noting the va... be found.

15. **Write down ... physical me... differential fo...**

⇒ Maxwell's equ...

i. **Gauss' law fo...**

$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$

According to ... through closed ... enclosed.

or, $\oint \vec{E}.d\vec{s}$ ...

Using divergenc...

or, $\int \nabla.\vec{E}\,dv =$ ...

or, $\nabla.\vec{E} = \frac{\rho}{\varepsilon_0}$

equation.

ii. **Gauss' law for m...**

$\oint \vec{B}.d\vec{s} =$ ...

Since magnetic l... infinity, the numb... arbitrary closed ...

$$\boxed{\text{or, } C = \frac{t_{1/2}}{R \log 2}}$$

Noting the value of $t_{1/2}$ and R , capacitance of capacitor can be found.

15. **Write down Maxwell equation in integral form with their physical meanings. Convert these equations into differential form.**

⇒ Maxwell's equation in integral form are

i. **Gauss' law for electrostatics**

$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0} \text{ ....(1)}$$

According to first equation, total electric flux passing through closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed.

or, $\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\int \rho \, dv \quad [\text{since } q = \int \rho dv]$

Using divergence theorem, $\oint \vec{A}.d\vec{s} = \int \nabla.\vec{A}\, dv$

or, $\int \nabla.\vec{E}\, dv = \frac{1}{\varepsilon_0}\int \rho \, dv$

or, $\nabla.\vec{E} = \frac{\rho}{\varepsilon_0}$ which is differential form of Maxwell's equation.

ii. **Gauss' law for magnetism**

$$\oint \vec{B}.d\vec{s} = 0$$

Since magnetic lines of force are either closed or go off to infinity, the number of magnetic lines of force entering any arbitrary closed surface is equal to lines of force leaving

from it. It means the flux of magnetic induction $\vec{B}$ across any closed surface is zero.

or, $\int \nabla.\vec{B}\, dv = 0$

As the surface bounding the volume is arbitrary, therefore this equation holds only if the integral vanish

i.e., $\nabla.\vec{B} = 0$

which is differential form of Maxwell's second equation.

iii. **Faraday's law of electromagnetic induction**

$$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$$

When magnetic field varies with time, an electric field will be produced. Here, electromotive force around a closed path is equal to the time derivative of the magnetic flux through any closed surface.

Using stoke's theorem to above equation

$$\oint \nabla \times \vec{E}.d\vec{s} = -\frac{\partial}{\partial t}\oint \vec{B}.d\vec{s}$$

i.e., $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

which is differential form of Maxwell's 3[rd] equation.

iv. **Ampere's law as extended by Maxwell**

$$\oint \vec{B}.d\vec{l} = \mu_0\left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

As soon as electric field is applied between plates of condenser, electrons of dielectric undergo a displacement within atom. This electronic movement constitute displacement current. Hence, the total current through the circuit will be sum of displacement current and conduction current.

$$\text{or, } (\nabla \times \vec{B}).d\vec{s} = \mu_0 \oint \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right].d\vec{s}$$

$$\therefore \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right)$$

This is the Maxwell's fourth equation in differential form.

16. **An electron is confined to an infinite height box of size 0.1 nm. Calculate the ground state energy of the electron. How this electron can be put to the third energy level?**

⇒ Mass of the electron (m) = 9.1×10^{-31} kg

$L = 0.1 \text{ nm} = 0.1\times10^{-9}$ m

$E_n = ?$

The permitted electron energies $= E_n = \frac{n^2 h^2}{8mL^2}$

$$= n^2 \frac{(6.626\times10\text{-}34)^2}{8\times9.1\times10^{-31}\times(0.\times10^{-9})^2\times1.6\times10^{-19}} = 38n^2 \text{ eV}$$

Ground state energy is $E_1 = 38$ eV

Energy of electron in 3rd level is $E_3 = 9E_1 = 342$ eV

Difference in energy is $E_3 - E_1 = (342 - 38)$ eV $= 304$ eV

As $E = \frac{hc}{\lambda}$

$$\text{or, } \lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{304\times1.6\times10^{-19}} = 4.08\times10^{-9} \text{ m}$$

By striking electron with photon of wavelength 408 nm, electron can be put into third energy level.

Important Numericals from Past IOE Exams

1. **A spring is hung vertically and loaded with a mass of ... g and allowed to oscillate. Calculate (i) time period ... frequency of oscillation when the spring is further loaded with 100 g producing an extension of 5 cm.**

⇒ Using the condition,

$$mg = kx$$

$$\text{or, } k = \frac{mg}{x}$$

$$\therefore k = \frac{0.1\times9.8}{0.05} = 19.6 \text{ N/m}$$

(i) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.075}{19.6}} = 0.39$ s

(ii) $f = \frac{1}{T} = \frac{1}{0.39} = 2.56$ Hz

20. **A thin straight, uniform rod of length $l = 1$ m and mass $m = 160$ gm hangs from a pivot at one end. (i) What is ... period for small amplitude oscillation? (ii) What is the length of a simple pendulum that will have the same period?**

⇒

(i) $T = 2\pi\sqrt{\frac{I}{mgl'}}$

$I = \frac{1}{3} m l^2$, $l' = \frac{l}{2}$, $g = 9.81 \text{ m/s}^2$

$\therefore T = 1.64$ s

(ii) Using the relation,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\text{or, } L = \frac{gT^2}{4\pi^2} = \frac{9.8\times(1.64)^2}{4\pi^2}$$

$\therefore L = 0.67$ m

3. **A source ... amplitude ... square cm ... and its dens...**

⇒ $\frac{E}{A\times t} = I$ ...

$= 2\pi^2$ ...

$= 1.3$ ...

4. **By how muc... change when i... that place is do...**

⇒ The intensity le...

$I_L = 10\log_{10}\left(\frac{I}{I_0}\right)$

We may write fo...

$I_{L1} = 10\log_{10}$ ...

$I_{L2} = 10\log_{10}$ ...

By the question,

$I_{L1} = 10\log_{10}$ ...

$I_{L2} = 10\log_{10}$ ...

Change in intens...

$I_{L2} - I_{L1} = 10 [\log$ ...

$= 10\log$ ...

$= 10\log$ ...

$= 3.01$

...rtically and loaded with a mass of 75 ...scillate. Calculate (i) time period (ii) ...tion when the spring is further loaded ...g an extension of 5 cm.

...6 N/m

$= 2\pi\sqrt{\frac{0.075}{19.6}} = 0.39$ s

... = 2.56 Hz

...rm rod of length l = 1 m and mass ...m a pivot at one end. (i) What is its ...plitude oscillation? (ii) What is the ...endulum that will have the same

$\frac{l}{2}$, $g = 9.81 m/s^2$

...on,

$\frac{...(1.64)^2}{4\pi^2}$

3. **A source of sound has a frequency of 256 Hz and amplitude of 0.50 cm. Calculate the energy flow across a square cm per sec. The velocity of sound in air is 330 m/s and its density is 1.29 kg/m³.**

⇒ $\frac{E}{A \times t} = I$

$= 2\pi^2 v f^2 a^2 = 2\pi^2 \times 330 \times 1.29 \times (256)^2 \times (0.5 \times 10^{-2})^2$

$= 1.38 \times 10^4 \ Jm^{-2}s^{-1}$

4. **By how much would intensity level at a given place change when intensity of sound produced by a source at that place is doubled?**

⇒ The intensity level of sound is expressed as

$I_L = 10 \log_{10}\left(\frac{I}{I_0}\right)$

We may write for two cases as

$I_{L1} = 10 \log_{10}\left(\frac{I_1}{I_0}\right)$

$I_{L2} = 10 \log_{10}\left(\frac{I_2}{I_0}\right)$

By the question, $I_2 = 2I_1$

$I_{L1} = 10 \log_{10}\left(\frac{I_1}{I_0}\right)$

$I_{L2} = 10 \log_{10}\left(\frac{2I_1}{I_0}\right)$

Change in intensity level is calculated as

$I_{L2} - I_{L1} = 10 \left[\log_{10}\left(\frac{2I_1}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right) \right]$

$= 10 \log_{10}\left(\frac{2I_1}{I_1}\right)$. $\qquad \because \log a - \log b = \log \frac{a}{b}$

$= 10 \log_{10} 2$

$= 3.01$ dB

5. **What is the reverberation time for a hall with length 12 m, breadth 11m, and height 9m if the coefficient of absorption of walls, ceiling, and floor are 0.02, 0.04, and 0.08 respectively?**

⇒ $V = 12 \times 11 \times 9 = 1188 \ m^3$, $\alpha_1 = 0.02$, $\alpha_2 = 0.04$, $\alpha_3 = 0.08$

Area of four walls $(S_1) = 2h(l + b) = 2 \times 9(12 + 11) = 414 m^2$

Area of ceiling(S_2) = area of floor $(S_3) = l \times b = 12 \times 11 = 132 m^2$

The reverberation time of hall is

$T = \frac{0.158V}{\Sigma_i \alpha_i S_i}$

$= \frac{0.158 \times 1188}{0.02 \times 414 + 0.04 \times 132 + 0.08 \times 132} = 7.78$ s

6. **Calculate the reverberation time of a small hall in BICC of 1500 m³ having seating capacity 120 persons when (i) the hall is empty, and (ii) with full capacity of the audience for the following data.**

Surface	Area	Coefficient of absorption
Plastered walls	112 m²	0.03
Wooden floor	130 m²	0.06
Plastered ceiling	170 m²	0.04
Wooden doors	20 m²	0.06
Cushioned chairs	120 m²	0.05
Audience	120	0.44

⇒ $V = 1500 \ m^3$

(i) For an empty hall,

$T = \frac{0.158V}{\Sigma_i \alpha_i S_i}$

$= \frac{0.158 \times 1500}{0.03 \times 112 + 0.06 \times 130 + 0.04 \times 170 + 0.06 \times 20 + 0.05 \times 120}$

$= 9.42$ s

(iii) For the hall with full capacity of audience,

$$T = \frac{0.158V}{\Sigma_i \alpha_i S_i}$$

$$= \frac{0.158 \times 1500}{0.03 \times 112 + 0.06 \times 130 + 0.04 \times 170 + 0.06 \times 20 + 0.05 \times 120 + 0.44 \times 120}$$

$= 3.04$ s

7. **In the Newton's ring experiment, the diameter of the tenth ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.**

⇒ Let μ be the refractive index of the liquid. If D_1 and D_2 be the diameter of tenth ring when the medium is air and liquid respectively, then

$$D_1^2 = 4n\lambda R$$

$$D_2^2 = \frac{4n\lambda R}{\mu}$$

$$\frac{D_1^2}{D_2^2} = \frac{4n\lambda R}{\frac{4n\lambda R}{\mu}}$$

$$\therefore \mu = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{1.4}{1.27}\right)^2 = 1.215$$

8. **An air wedge of angle 0.01 radians is illuminated by monochromatic light of wavelength 600 nm falling normally on it. At what distance from the edge of the wedge will the 12th fringe be observed by reflected light?**

⇒ $\alpha = 0.01$ radian, $\lambda = 600 \times 10^{-9}$ m, $n = 12$

We have,

$$x = \frac{n\lambda}{2\alpha} = \frac{12 \times 600 \times 10^{-9}}{2 \times 0.01} = 3.6 \times 10^{-4} \text{ m}$$

9. **Newton's rings formed by sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of 40th ring?**

⇒ For dark fringe, we have

$$D_n^2 = 4n\lambda R$$

For $n = 40$, $D_{40}^2 = 4 \times 40\lambda R$ .... (i)

Let 'm' be the order of the dark ring which will have double the diameter of 40th ring. Then, we can write

$(2 \times D_{40})^2 = 4m\lambda R$ .... (ii)

Dividing equation (i) by (ii), we get

$$\frac{1}{4} = \frac{40}{m}$$

$$\therefore m = 160$$

10. **Newton's rings are formed by reflected light of wavelength 5895 A° with a liquid between the plane and curved surfaces. If the diameter of the sixth bright ring is 3 mm and the radius of the curved surface is 100 cm, calculate the refractive index of the liquid.**

⇒ $\lambda = 5895 \times 10^{-10}$, $D_6 = 3 \times 10^{-3}$, $R = 100 \times 10^{-2}$ m

For bright fringe, we have\

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$$\text{or, } D_6^2 = \frac{2(2 \times 6 - 1) \times 5895 \times 10^{-10} \times 100 \times 10^{-2}}{\mu}$$

$$\text{or, } (3 \times 10^{-3})^2 = \frac{22 \times 5895 \times 10^{-10}}{\mu}$$

$$\therefore \mu = 1.2$$

11. **A wedge shaped air film having an angle 45'30" is illustrated by monochromatic light and fringes are observed normally. If the fringe width is 0.12 cm, calculate the wavelength of light used.**

⇒ $\beta = 0.12 \times 10^{-2}$, $\alpha = 45'30'' = 45.5 \times \frac{\pi}{2}$ radians

We have,

$$\beta = \frac{\lambda}{2\alpha}$$

..............(i)

k ring which will have double

we can write

...(ii)

ve get

d by reflected light of

uid between the plane and

r of the sixth bright ring is

curved surface is 100 cm,

f the liquid.

100×10^{-2} m

$\times10^{-2}$

g an angle 45'30" is

ght and fringes are

width is 0.12 cm,

d.

dians

or, $\lambda = 2\alpha\beta$

$\therefore \lambda = 2\times45.5\times\frac{\pi}{2}\times0.12\times10^{-2} = 0.171$ m

12. **A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines per cm and the third order spectral line is found to be diffracted through 35°. Calculate the wavelength of light.**

⇒ $a+b = \frac{1}{N} = \frac{1}{500}$ cm, $n = 3$

Using the relation,

$$(a+b)\sin\theta = n\lambda$$

or, $\frac{1}{500}\times\sin 35° = 3\lambda$

$\therefore \lambda = 3.824\times10^{-5}$ cm $= 3824$ A°

13. **What is the highest order spectrum which may be seen with monochromatic light of wavelength 559 nm by means of diffracting grating with 15000 lines per inch?**

⇒ $(a+b) = \frac{1}{N} = \frac{1}{15000}$ inch $= \frac{1}{15000}\times2.54\times10^{-2} = 1.693\times10^{-6}$

$\lambda = 559\times10^{-9}$ m

We have,

$$(a+b)\sin\theta = n\lambda$$

For the highest order spectrum, $\sin\theta = 1$

or, $(a+b) = n\lambda$

or, $1.693\times10^{-6} = n\times559\times10^{-9}$

$\therefore n \approx 3$

14. **Calculate the polarizing angle of light travelling from water ($\mu_1 = 1.33$) to glass ($\mu_2 = 1.5$).**

⇒ $\theta_P = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \tan^{-1}\left(\frac{1.5}{1.33}\right) = 48.44°$

15. **What is the capacitance of cylindrical capacitor made of two axial cylinders of length 2 cm and radii 2 mm and 2.1 mm? The space between the cylinders is filled with a medium of relative permittivity 7.8.**

⇒ $L = 2\times10^{-2}$ m, $a = 2\times10^{-3}$ m, $b = 2.1\times10^{-3}$ m, $\varepsilon_r = 7.8$

We have,

$$C = 4\pi\varepsilon\frac{L}{\ln\left(\frac{b}{a}\right)} = 4\pi\varepsilon_0\varepsilon_r\frac{L}{\ln\left(\frac{b}{a}\right)}$$

$$= 4\pi\times8.854\times10^{-12}\times7.8\times\frac{2\times10^{-2}}{\ln\left(\frac{2.1\times10^{-3}}{2\times10^{-3}}\right)}$$

$$= 3.56\times10^{-16}\text{ F}$$

16. **What is the capacitance of a spherical capacitor made of two concentric spheres of radii 6 and 6.01 cm? The space between them is filled with a medium of relative permittivity equal to 8.**

⇒ $a = 6\times10^{-2}$ m, $b = 6.1\times10^{-2}$ m, $\varepsilon_r = 8$

We have,

$$C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right) = 4\pi\varepsilon_0\varepsilon_r\left(\frac{ab}{b-a}\right)$$

$$= 4\pi\times8.854\times10^{-12}\times8\times\left(\frac{6\times10^{-2}\times6.1\times10^{-2}}{6.1\times10^{-2}-6\times10^{-2}}\right)$$

$$= 3.26\times10^{-9}\text{ F}$$

17. **A variable field 10^{12} V/ms is applied to a parallel plate capacitor with circular plates of diameter 10 cm. Calculate induced magnetic field and displacement current.**

⇒ $\frac{dE}{dt} = 10^{12}$ V m^{-1}s^{-1}, $d = 10\times10^{-2}$ m, $r = 5\times10^{-2}$ m

We have,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\phi}{dt}$$

$$\text{or, } 2\pi r B = \mu_0 \varepsilon_0 (\pi r^2) \frac{dE}{dt}$$

$$\text{or, } B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}$$

$$\therefore B = 2.78 \times 10^{-7}\, T$$

Displacement current is

$$i_d = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

$$= 8.854 \times 10^{-12} \times \pi (5 \times 10^{-2})^2 \times 10^{12}$$

$$= 6.95 \times 10^2\, A$$